Performance Analysis of Two-Hop Cooperative MIMO Transmission with Best Relay Selection in Rayleigh Fading Channel

Ahasanun Nessa, Qinghai Yang, and Kyung-sup Kwak
Graduate School of IT and Telecommunications, Inha University, South Korea
State Key Laboratory of ISN, School of Telecom, Engineering, Xidian University

Abstract: *Wireless Relaying is a promising solution to overcome the channel impairments and provides high data rate coverage that appears for beyond 3G mobile communications. In this paper we present end to end BER performance of dual hop wireless communication systems equipped with multiple Decode and Forward relays over Rayleigh fading channel with the best relay selection. We compare the BER performance of the best relay with the BER performance of single relay. We select the best relay based on the end to end channel conditions. We further calculate the outage probability of the best relay. It is shown that the outage probability of the best relay is equivalent to the outage probability when all relays take part in the transmission. We apply orthogonal space time block coding at the source terminal. Numerical and simulation results are presented to verify our analysis.*

Keywords: *Bit error rate, amplify and forward, multiple input multiple output, decode-and-forward, and probability density function (pdf).*

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1. Introduction

Dual hop transmission is a technique by which the channel from source to destination is split into two shorter links using a relay [4]. In this case the key idea is that the source relays a signal to destination via a third terminal that acts as a relay. It is an attractive technique when the direct link between the base station and the original mobile terminal is in deep fade or heavy shadowing or there is no direct link between source and destination.

On the other hand diversity technique is an effective technique to mitigate the severe form of interference that arises due to the multi path propagation of wireless signal gain without increasing the expenditure of transmission time or bandwidth. Although transmission diversity is clearly advantageous on a cellular base station, it might not be practical for other scenarios. Specially due to cost, size or hardware limitations, a wireless device may not be able to support multiple transmission antennas. In order to overcome this limitation a new form of diversity technique, the cooperative diversity (named so as it comes from user cooperation) has been introduced by [8, 9, 11, 14]. It exploits the broadcast nature of the wireless transmission. In cellular, the ad-hoc network when one user is transmitting information to a remote terminal, other users nearby also receive it and transmit the signal to the destination. This process results in multiple copies of same signal from independent fading paths at the destination and brings diversity.

Depending on the nature and the complexity of the relays cooperative transmission system can be classified into two main categories; regenerative and non regenerative systems. In regenerative systems, relay fully decodes the signal that went through the first hop. Then retransmits the decoded version to the second hop. This is also referred to as decode- and-forward or digital relaying. On the other hand, non regenerative systems use less complex relays that just amplify and forward the incoming signal without performing any sort of decoding. It is called amplify-and-forward [9] or analog [15] relaying. The performance of both systems has been well studied in [3, 4, 5, 7]. In [10] a distributed Space Time Coded (STC) cooperative scheme is proposed, where the relays decode the received symbols from the source and utilize a distributed STC. The number of distributed antennas (distributed relays) for cooperation is generally unknown and also may be not unique. So coordination among the cooperating nodes is needed prior to the use of a specific space time coding scheme.

Moreover, if many relay stations transmit signal to destination then it also needs synchronization of carrier phases among several transmit receive pairs which will increase the complexity of receiver as well as cost. Choosing the minimum number of relays for reducing cooperation overhead and saving energy without performance loss is an important concern. There are various protocols proposed to choose the best relay among a collection of available relays in literature. In [16], the author proposed to choose the best relay depending on its geographic position, based on the geographic random forwarding protocol proposed by [18, 17]. In [2], the author proposed opportunistic relay based on the instantaneous channel conditions. This single relay opportunistic selection provides no performance loss from the perspective of diversity-multiplexing gain trade off, compared to schemes that rely on distributed space time coding [14].

In this paper our aim is to analyze the system with multiple relay nodes where source has two transmit antennas and each relay and destination have one antenna. In the second hop, before transmitting signal to destination the best relay is selected based on the instantaneous channel conditions of two hops. This technique can save the transmission power of the network. It also reduces the decoding complexity at receiver side and at the same time achieves diversity gain. However this intermediate relay shall increase the maximum distance between source and destination also increase the spectral efficiency.

The paper is outlined as follows: section 2 introduces channel model. Relay selection protocol is described in section 3. Section 4 derives the Probability Density Function (PDF) of the received SNR per bit and analyzes the BER performance of the best relay. Section 5 shows the outage behaviour of the best relay. Simulation results are presented in Section 6 and finally section 7 presents conclusion and future work.

2. System and Channel Model

We are considering a wireless dual hop network where a number of relay nodes are placed randomly and independently according to some distribution. The direct link between source and destination may be blocked by some obstacles. The relays can communicate with both end points. In our model the source equipped with two transmit antennas and each relay node has a single antenna which can be used for both transmission and reception. All transmissions are assumed to be half duplex and therefore a relay station can not transmit and receive at the same period. During the first hop source broadcasts symbols, the relays listen and during the second hop relays forward the decoded version of the received signal to destination.

Figure 1 shows the channel model. We are assuming the channel remains constant during the two hops with Rayleigh fading. We are applying OSTBC at the source. No channel information is available at source. So no power or bit loading is performed at source. Each transmission antenna of source is assumed to use the same transmission power $\sigma_s^2 = P/t$, where P is the total transmission power of the base station and t is the number of antennae at base station. In this paper we are considering t=2. For two transmit antenna, there exists a rate one OSTBC defined by the transmission matrix X,

$$X = \begin{pmatrix} X_1 & -X_2^* \\ X_2 & X_1^* \end{pmatrix} \qquad (1)$$

where x_1 and x_2 are a pair of complex symbols to be transmitted and * denotes the complex conjugate.

We assume there are r relays and number of transmit antennae at source is 2. So the channel matrix for the first hop is given by

$$H_{SR} = \begin{pmatrix} h_{11} & h_{12} \\ \vdots & \vdots \\ h_{r1} & h_{r2} \end{pmatrix} \qquad (2)$$

where the element h_{ij} denotes the channel gain between the i th relay and the j th transmission antenna of source, $i = 1, 2, ...r$ and $j = 1, 2$ We assume that each element of H_{SR} is independent and identically distributed complex Gaussian random variable with zero mean and β_1 variance. If we notice carefully we observe each row of H_{SR} represents the channel coefficient between source and relay. So the channel matrix for each relay can be represented by

$$\alpha_i = \begin{pmatrix} h_{i1} & h_{i2} \end{pmatrix} \quad for\ i= 1,\ 2,\ ...r \qquad (3)$$

And for the second hop g_i is the individual relay to destination fading amplitude.

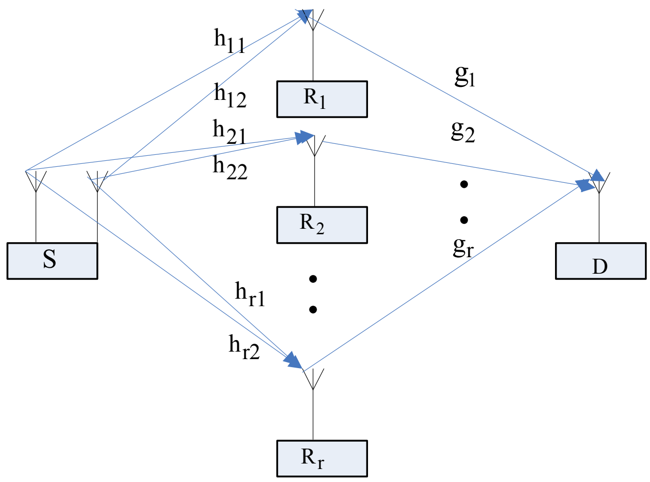

Figure 1. A half duplex dual hop relay network.

3. Relaying Protocol

In single relay selection schemes only one opportunistic relay transmits the received signal to destination. In previous work opportunistic relay is defined considering distance toward source or destination [17] or considering the channel condition. The selection of the best relay based on distance is not a good selection since communication link between transmitter and receiver locating in the same distance might have enormous difference in terms of received signal due to fading and shadowing. In [2], authors select the best relay based on the channel condition. In this paper we are assuming all relays can listen to each other. After monitoring the instantaneous channel condition each relay also broadcast the information to each other. If the relays are hidden to each other then they send information to destination and destination decides among them which is the most opportunistic for relaying and broadcasts the decision to relay stations [2].

Let α_{si} and α_{id} denote the total channel power from source to ith relay and ith relay to destination respectively. Here, α_{si} and α_{id} describe the quality of the wireless path between source to relay and destination to relay. α_{si} is calculated by relay i by the following equation.

$$\alpha_{si} = |h_{i1}|^2 + |h_{i2}|^2 \quad . . \tag{4}$$

and $\alpha_{id} = |g_i|^2$ is the fading amplitude from relay to destination. Since the two hops are both important for end to end performance, each relay calculates corresponding h_i based on the two decision rules.

Rule1: $$h_i = \min\{\alpha_{si}, \alpha_{id}\}. \tag{5}$$

Rule 2:

$$h_i = \frac{2}{\frac{1}{\alpha_{si}} + \frac{1}{\alpha_{id}}} = \frac{2\alpha_{si}\alpha_{id}}{\alpha_{si} + \alpha_{id}} \tag{6}$$

The relay i that maximizes function h_i is one with the "best" end to end path between initial source to destination. After being selected as the best relay it relays signal to destination. In this paper it is assumed the destination have perfect channel information available for decoding the received signal.

3. End to-End Ber Analysis

In the half duplex two hop protocol during the first time slot, the source transmits while all the relay nodes listen and during the next time slot the best relay is selected based on equations 5 and 6, then it transmits to destination .The selected relay is most opportunistic among all pairs for relaying signal to destination. The end to end SNR through this selected relay i is given by

$$\gamma = \max_{i \in 1,..r}(\min(\gamma_{si}, \gamma_{id})) \tag{7}$$

where γ_{si} and γ_{id} are the instantaneous SNR of the S-R and R-D link, respectively.

The selection of the best relay is done by order statistics. The first step is to obtain the weaker link between the first hop and the second hop of each relay node. The weak link is ordered and the one with the largest SNR is selected as the candidate relay to perform detection and forward to destination. We assume the S-R and R-D link have the same average channel gain. The probability density function of γ can be obtained as [1].

$$f(\gamma) = 2rf(\gamma^*)(1 - F(\gamma^*))(2F(\gamma^*) - F(\gamma^*)^2)^{r-1} \tag{8}$$

where

$$f(\gamma^*) = \frac{1}{\overline{\gamma^*}}\exp(-\frac{\gamma^*}{\overline{\gamma^*}})$$

and

$$F(\gamma^*) = 1 - \exp(-\frac{\gamma^*}{\overline{\gamma^*}})$$

are the pdf and cdf of Raleigh distributed random variable, respectively. Finally the pdf of γ can be obtained as

$$f(\gamma) = r\frac{\exp(-\frac{\gamma^*}{\overline{\gamma^*}/2})}{\overline{\gamma^*}/2}\left(1 - \exp(-\frac{\gamma^*}{\overline{\gamma^*}/2})\right)^{r-1} \tag{9}$$

and through the binomial expansion, we further can write [3]

$$f(\gamma) = \sum_{i=1}^{r}(-1)^{i-1}\binom{r}{i}\frac{2i}{\overline{\gamma^*}}\exp\left(-i\frac{2\gamma^*}{\overline{\gamma^*}}\right) \tag{10}$$

The pdf obtained in equation 10 can be employed for evaluating the error performance of this relaying scheme with any modulation techniques. BER for BPSK constellation.

$$\begin{aligned} P_{BPSK} &= \frac{1}{2}\int_0^\infty erfc(\sqrt{\gamma})f(\gamma)d\gamma \\ &= \frac{1}{2}\int_0^\infty erfc\left(\sqrt{\gamma}\right)\sum_{i=1}^{r}(-1)^{i-1}\binom{r}{i}\frac{2i}{\overline{\gamma^*}}\exp\left(-i\frac{2\gamma^*}{\overline{\gamma^*}}\right)d\gamma \end{aligned} \tag{11}$$

The PDF of phase θ of the received signal with SNR γ per bit is given as in [13].

$$f_\theta(\theta|\gamma) = \frac{1}{2\pi}e^{-\gamma\log_2 M}\left[1 + \cos\theta\sqrt{4\pi\log_2 M\,\gamma}\,e^{\gamma\log_2 M\cos^2\theta} \times\left(1 - \frac{1}{2}erfc\left(\sqrt{\gamma\log_2 M}\cos\theta\right)\right)\right] \tag{12}$$

where $\mathrm{erfc}(x) = \frac{2}{\sqrt{\pi}}\int_x^\infty e^{-y^2}dy.$ Then the exact probability that the phase θ of the received signals lies in a decision region $[\theta_l, \theta_u]$ is given by,

$$\Pr\{\theta \in [\theta_l, \theta_u]; \gamma\} = \int_0^\infty\int_{\theta_l}^{\theta_u} f_\theta(\theta|\gamma)f(\gamma)d\theta d\gamma\ . \tag{13}$$

Substituting equation 11 and 13, the BER of M-ary PSK constellation is as follows;

$$P_{M-PSK}(\gamma) = \frac{1}{\log_2 M}\sum_{j=1}^{M}e_j\Pr\{\theta \in \theta_j; \gamma\} \tag{14}$$

where $\theta_j = [(2j-3)\pi/M, (2j-1)\pi/M]$ for j=1,…M and e_j is the number of bit errors in the decision region θ_j.

4. Outage Probability

The mutual information between the source and relay nodes i= 1,2,…r in the first hop can be given by

$$I_{i1} = \frac{1}{2}\log(1+\Omega_{i1}SNR). \tag{15}$$

with $\Omega_{i1} = \frac{|h_{i1}|^2 + |h_{i2}|^2}{2}$. Ω_{i1} is exponential distribution with parameter $\lambda_1 = \frac{1}{2}$ as shown in Appendix A and the mutual information in the second hop of this corresponding relay is given by

$$I_{i2} = \frac{1}{2}\log(1+\Omega_{i2}SNR). \tag{16}$$

The probability density function of Ω_{i1} and Ω_{i2} are in order as follows.

$$f(\Omega_{i1},\lambda_1) = \lambda_1 e^{\Omega_{i1}\lambda_1} \tag{17}$$

$$f(\Omega_{i2},\lambda_2) = \lambda_2 e^{\Omega_{i2}\lambda_2} \tag{18}$$

So the capacity of the network for relay *i* is the minimum of the mutual information of this two hop.

$$C(\gamma_i) = \min(I_{i1}, I_{i2}). \tag{19}$$

We are selecting the best relay based on end to end channel condition. So the maximum capacity of the entire network depends on the mutual information of the best relay. The mutual information of the best relay can be given by

$$I = \max_{i\in|1,..,r|}(\min(I_{i1}, I_{i2})). \tag{20}$$

$$I = \max_{i\in|1,..,r|}(\min(\frac{1}{2}\log(1+\Omega_{i1}SNR, \frac{1}{2}\log(1+\Omega_{i2}SNR))). \tag{21}$$

So the network capacity

$$C(\gamma) = I \tag{22}$$

The outage probability P_{out} which can be defined as the probability that instantaneous capacity $C(\gamma)$ fall below outage capacity C_{out}.

$$P_{out} = \Pr(C(\gamma) < C_{out}) \tag{23}$$

$$P_{out} = \Pr(\max_{i\in|1,..,r|}(C(\gamma_i) < C_{out})) \tag{24}$$

Due to the independent channel assumption it given by

$$P_{out} = \prod_{i=1}^{r} P_{out}^{i} \tag{25}$$

with $P_{out}^{i} = \Pr(C(\gamma_i) < C_{out})$.

$$P_{out}^{i} = \Pr(\min(\frac{1}{2}\log(1+\Omega_{i1}SNR), \frac{1}{2}\log(1+\Omega_{i2}SNR)) < C_{out})$$
$$= \Pr(\min((\Omega_{i1},\Omega_{i2}) < w)), w = \frac{2^{2C_{out}}-1}{SNR} \tag{26}$$

Then by using order statistics in [18].

$$\Pr(\min(\Omega_{i1},\Omega_{i2}) = F(\lambda_1) + F(\lambda_2) - F(\lambda_1)F(\lambda_2)$$
$$= (1-e^{-\lambda_1 w}) + (1-e^{-\lambda_2 w}) - (1-e^{-\lambda_1 w})(1-e^{-\lambda_2 w})$$
$$= 1 - e^{-(\lambda_1+\lambda_2)\frac{2^{2C_{out}}-1}{SNR}} \tag{27}$$

$$P_{out}^{i} = 1 - e^{-(\lambda_1+\lambda_2)\frac{2^{2C_{out}}-1}{SNR}}. \tag{28}$$

and Finally by putting this value of P_{out}^{i} into equation 25 we get the outage probability P_{out}.

6. Simulation Result

In this section we are presenting our simulation results of BER performance and outage behavior. We consider BPSK and QPSK constellation for 2 transmit antennas equipped at source.

We are assuming that the channels are slow Rayleigh fading channel. Two sorts of simulation are performed, one for decision rule 1 and another for decision rule 2. We can see that the performances are nearly the same for both cases. From Figures 2 and 3 we see the BER performance of the best relay among a set of relays is always better than the BER performance of single relay. It is also shown that the better BER performance can be achieved by adopting more relay nodes. Modulation order also affects the difference between the BER performances. However, for higher modulation order, the difference becomes negligible. Comparing Figures 2 and 3 it is easily noticeable.

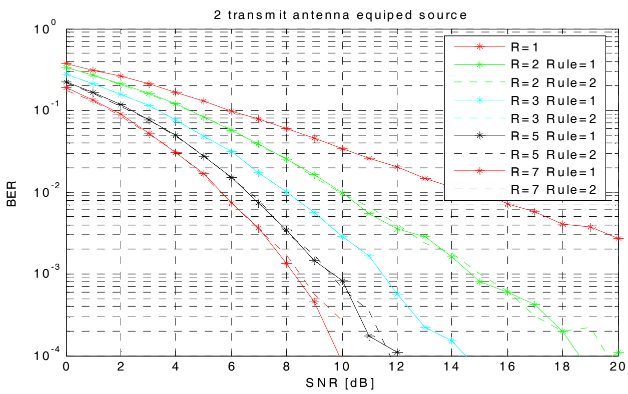

Figure 2. BER vs SNR with Relay selection. Tx=2, modulation= BPSK.

Figures 4 and 5 show the outage performance of the best relay for outage capacity C_{out} =.5 bps/Hz and C_{out} =1 bps/Hz, respectively. It is shown that theoretically result match with simulation result perfectly. Both figures show that the outage probability decreases with increase in number of relays. By adopting more relays better outage performance can be achieved.

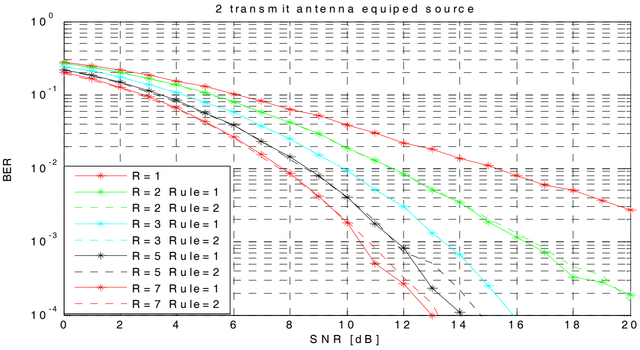

Figure 3. BER vs SNR with Relay selection. Tx=2, modulation =QPSK.

7. Conclusions

In this paper we presented end to end BER performance and outage performance of dual hop wireless transmission by selecting the best relay based on the instantaneous channel condition. Both BER performance and outage performance can be improved by adopting more relays. However the outage performance of the best relay is equivalent to the outage behaviour when all relay nodes participate into the second hop. The single relay selection can reduce receiver complexity and at the same time will increase the network coverage. In future we will continue our work in multi-hop transmission for covering long distance environment.

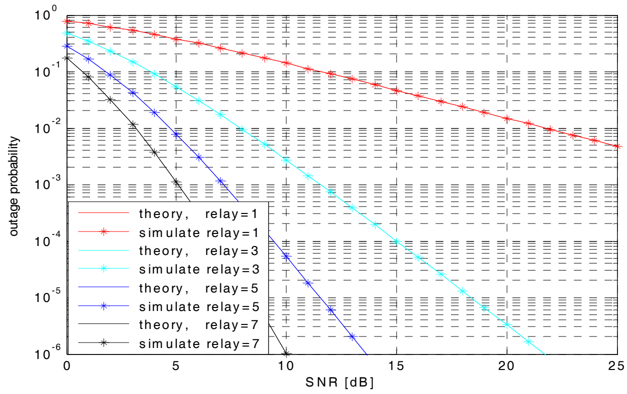

Figure 4. Outage probability vs SNR, C_{out} = .5 bps/Hz.

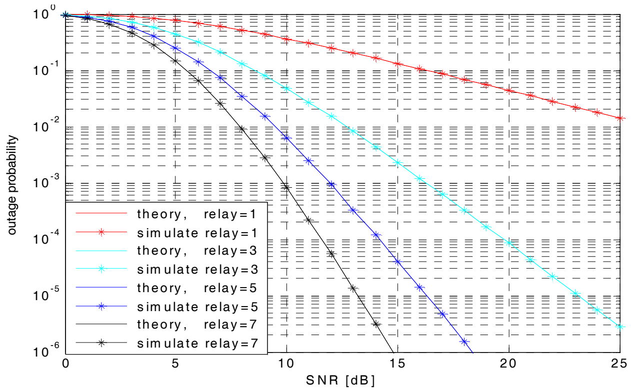

Figure 5. Outage probability vs SNR (db), C_{out} = 1 bps/Hz.

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Ahsananun Nessa received her BSc degree in computer science and engineering from Jahangirnagar University of Bangladesh. She received her MEng degree in IT and telecommunications engineering at INHA University, South Korea. In 2009, she joined as a faculty in Department of Computer Science and Information Technology, Patuakhali Science and Technology University, Bangladesh. Now she is pursuing her PhD in Department of Electrical and Computer Engineering at Concordia University, Canada. Her research interests include wireless communications emphasize on next generation networks and telecommunications diffusion in developing countries.

Qinghai Yang received his BS degree in communication engineering from Shandong University of Technology, China in 1998, MSc degree in information and communication systems from Xidian University, China, in 2001, and PhD in communication engineering from Inha University, Korea, with university-president award in 2007. From March 2007 to February 2008, he was a research fellow at UWB-ITRC, Korea. Now, he is with Xidian University as an associate professor. His current research interest lies in the fields of distributed communication and control, resource sharing in dynamic spectrum access networks, and MIMO UWB systems.

Kyung-sup Kwak received the MS degree from the University of Southern California in 1981 and the PhD degree from the University of California at San Diego in 1988. From 1989 to 1990 he was with the IBM Network Analysis Center at Research Triangle Park, North Carolina. Since then he has been with the School of Information and Communication, Inha University, Korea as a professor. In 2008, he is elected as Inha Fellow Professor (IFP). He published more than 100 SCI journal papers, more than 50 patents. His research interests include multiple access communication systems, UWB radio systems and WPAN/WBAN, sensor networks.

Appendix A

Let n=1,2..k be normally distributed random variable with mean 0 and variance 1, Then the random variable

$$Q \sim \sum_{i=1}^{k} X_i^2 \tag{29}$$

is distributed according to the chi-square distribution with k degrees of freedom. This pdf of Q is usually given by

$$f(x;k) = \begin{cases} \frac{1}{2^{k/2}\Gamma(k/2)} x^{(k/2)-1} e^{-x/2} & \text{for} \quad x > 0, \\ 0 & \text{for} \quad x \le 0, \end{cases} \tag{30}$$

when Γ is the gamma function defined by

$$\Gamma(\alpha) = \int_0^{\infty} x^{\alpha-1} e^{-x} dx \tag{31}$$

if k=2, then equation 30 be an exponential distribution with parameter ½.If k is more than 2 then it is a chi square distribution or gamma distribution [13].