Comparing Performance Measures of Sparse Representation on Image Restoration Algorithms

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Abstract: *Image restoration is a systematic process that regains the lost clarity of an image. In the past, image restoration based on sparse representation has resulted in better performance for natural images. Within each category of image restoration such as de-blurring, de-noising and super resolution, different algorithms are selected for evaluation and comparison. It is evident that both local and non-local methods within each algorithm can produce improved image restoration results based on the over complete representations using learned dictionary. The Gaussian noise is added with the original image and comparative study is made from the three different de-noising techniques such as mean filter, Least Mean Square (LMS) adaptive filters and median filters. The experimental results arrived from the filters are discussed for each model of the selected image restoration algorithms-locally adaptive sparsity and regularization, Centralized Sparse Representation (CSR), low-rank approximation structured sparse representation and non-locally CSR. A comprehensive study of this paper would serve as a good reference and stimulate new research ideas in Image Restoration (IR).*

Keywords: *IR, sparse representation, image de-blurring, locally adaptive sparsity, CSR.*

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1. Introduction

Image Restoration (IR) intends to recover high resolution image from low resolution image. Degradation in an image caused due to blur, noise and atmospheric turbulence can be removed to regain the original image. Blurring is a process of reducing the bandwidth of an ideal image that results in imperfect image formation. It happens due to the relative motion between the camera and the original scene or by atmospheric turbulence and relative motion between camera and ground. IR concerned with the estimation or reconstruction of uncorrupted image from a blurred or noise one.

In addition to these blurring effects, noise also corrupts any recorded image. IR can be modelled by the system as shown in Equation 1.

$$y = Hx + v \tag{1}$$

Where $x∈R^N$ is the unknown high quality original image, $H∈R^{M\times N}$ is the degradation matrix, $v∈R^N$ is the additive noise and y is the observed measurement. When H is specified by Kernel, then image reconstruction is the problem of image blurring.

The solution for the de-blurring problem can be obtained by solving the optimization problem as shown by Equation 2.

$$\hat{x} = \underset{x}{argmin}\left\{\| y - Hx \|_2^2 + \lambda . J(x)\right\} \tag{2}$$

Different types of filters are used to restore the image from noisy and blur images. In image restoration, the improvement in quality of the restored image over the recorded blurred one is measured by the signal-to-noise-ratio improvement.

In the past decades, different methods and filters have been used for the purpose of image restoration. These methods do not hold to be proven to restore the image in case of additive white noise and Gaussian noises. Sparse representations approximate an input vector by using a sparse linear combination of atoms from an over complete dictionary. Sparse based methods have been verified to perform well in terms of Mean Square Error (MSE) measure as well as Peak Signal-to-Noise Ratio (PSNR). Sparse based models are used in various image processing fields such as image de-noising, image de-blurring, super resolution, etc.

In the sparse model of the IR, sparse coefficients should be close to that of the unknown original image in order to recover sharp edges. Sparse representation over local dictionary uses the image local patch. Image substance can differ drastically across the image, it is necessary to acclimatize the dictionary to each local patch. This representation can be done over a selected dictionary D for a set of patches. These patches on the D are estimated to obtain reconstruct image.

In this paper, we attempt to compare the three different de-noising techniques for Gaussian noise integrated image and four different image restoration methods based on sparse representation. This paper is organized as follows: Section 2 presents the related work on sparse representation and filter techniques. Section 3 provides the different methodologies of IR along with sparse representation. Section 4 presents the experimental results and section 5 concludes the paper along with the future scope of research in this specific area.

2. Related Work

Biemond *et al.* [2] discuss the iterative restoration algorithms for the elimination of linear blur from images that are tainted by pointwise nonlinearities such as additive noise and film saturation. There are various basic iterative solutions such as inverse filter solution, least squares solutions, wiener solution, constrained least squares solution, kalman filter solution. Inverse filter is a linear filter whose point spread function is the inverse of blurring function. It requires only the blur point spread function. Least square filters are used to overcome the noise sensitivity and weiner filter is a linear partial inverse filter which minimizes the MSE with the help of chosen point spread function. Constrained least squares filter for overcoming some of the difficulties of inverse filter and of wiener filter and it also estimates power spectrum. Regularization methods associated with the names of Tikhonov and Miller. For both non-iterative and iterative restorations based on Tikhonov-Miller regularization analysed using eigen vector expansions.

Iterative image restoration algorithms [3] are admired methods for de-convolving images from satellite sensors and medical tomography. These methods are non-linear and it suggests more effective restoration than simple techniques such as linear inverse filtering which often not succeed. Non-linear methods are useful when the data is noisy or imperfect which the case in practical applications is usually. Iterative techniques are operated on the result of the previous iteration and are normally slow to come towards the final outcome. They give better control and enhanced results compared with linear methods, but have significant computational requirements.

Sparse representation of image signals declares a sparse decomposition over a redundant dictionary for handling sources of data. The problems of learning dictionaries for color images and extend the Singular Value Decomposition (K-SVD) based grayscale image de-noising algorithm was described by Elad and Aharon (2006). Marial *et al.* [10] promote the work for handling non homogenous noise and missing information in application such as colour image de-noising, demosaicking and in-painting. Sparse land model suggests dictionaries for various classes of signals and the sparsity of signal decomposition is a powerful model. The removal of additive white Gaussian noise with gray-scale images make use of the K-SVD for learning the dictionary from the noisy image directly. The extension to colour can be easily performed by simple concatenation of the RGB values to the single vector and training on those directly which gives better results than de-noising each channel separately.

Chatterjee and Milanfar [4] proposed Locally Learned Dictionaries (K-LLD): A patch based locally adaptive de-noising method based on clustering results in region of similar geometric structure from the given noisy image using K-LLD. The features of local weight function derived from steering regression are utilized for the formation of clusters. With the help of kernel regression, dictionary estimates the underlying pixel values and Stein Unbiased Risk Estimator (SURE) estimates local patch size for chosen images. Kernel regression framework uses the methods such as bilateral filter, nonlocal means and optimal spatial adaptation. De-noising can be learned with a suitable basis function that describes geometric structure of image patches. Image de-noising can be first performed by explicitly segmenting the image based on local image structure and through efficient data representation [5].

Clustering based de-noising (K-LLD) makes use of locally learned dictionary that involves clustering, dictionary selection and co-efficient calculation. Iterative de-noising is also done to improve the final de-noised image. After segmented the image, kernel regression is performed to finally de-noise the image. Since the de-noising is done mainly based on the clustering stage, if the number cluster is large this method is not sensitive. In this case, it may be useful to use variants of K-means that converge to most favourable number of clusters automatically.

Aharon *et al.* [1] address the image de-noising problem zero-mean white and homogenous Gaussian additive noise is to be isolated from the given image. Based on sparse and redundant representation over trained dictionaries, image content dictionaries are obtained using K-SVD algorithm. Using corrupted image or high quality image database training is done. So far, K-SVD algorithm is used to handle small image patches we extend it to handle large image patches. Sparsity of unitary wavelet coefficient was considered leading to shrinkage algorithm. Basic pursuit and matching pursuit de-noising give raise the ability to address image de-noising problem as a direct sparse decomposition technique over redundant dictionaries. This work focus on the small image patches on global structure of the image and cannot be directly deployed on larger blocks.

3. Present Methodology

3.1. Filters

A mean filter acts on an image by smoothing it by reducing the intensity variation between adjacent pixels. The mean or average filter works on the shift-multiply-sum principle. An adaptive filter does a better job of de-noising images compared to the averaging filter. The fundamental difference between the mean filter and the adaptive filter lies in the fact that the weight matrix varies at iterations in the adaptive filter while it remains constant throughout the iterations in the mean filter. Adaptive filters are capable of de-noising non-stationary images, that is, images that have abrupt changes in intensity. Such filters are known for their ability in automatically tracking an unknown circumstance or when a signal is variable with little a priori knowledge about the signal to be processed. In general, an adaptive filter iteratively adjusts its parameters during scanning the image to match the image generating mechanism. This mechanism is more significant in practical images, which tend to be non-stationary.

Compared to other adaptive filters, the Least Mean Square (LMS) adaptive filter is known for its simplicity in computation and implementation. The basic model is a linear combination of a stationary low-pass image and a non-stationary high-pass component through a weighting function. Thus, the function provides a compromise between resolution of genuine features and suppression of noise. A median filter belongs to the class of nonlinear filters that follows the moving window principle as same as mean filter. The median of the pixel values in the window is computed, and the centre pixel of the window is replaced with the computed median. Median filtering is done by, first sorting all the pixel values from the surrounding neighbourhood into numerical order and then replacing the pixel being considered with the middle pixel value.

Sa'dah *et al.* [11] discussed in image enhancement that low pass filters blur the images which result in noise reduction, where as high pass filters used to sharpen the images. Butterworth filter and Gaussian filter can be used to sharpen the images and also high pass filter reside in the shape of the curve. Therefore, any one of the high pass filters can be used to sharpen the images in restoration algorithm.

3.2. Locally Adaptive Sparsity and Regularization (LASR)

Sparse representation finds a space where the local image patch exhibits high sparsity and to find out the image local sparsity. To find the locally varying sparsity [8], it is necessary to locally adapt the dictionary learning process and the sparsity-regularization parameters. The sparsity regularization parameters λ are locally estimated for each co-efficient and updated along with adaptive learning dictionaries Principle Component Analysis (PCA). Assume that the observation y is contaminated with additive Gaussian noise with standard deviation σ_n. Under Bayesian framework, the sparsity vector can be modeled as Equation 3.

$$\hat{\alpha} = \underset{\alpha}{argmax}\left\{logp(\alpha/y)\right\} = \underset{\alpha}{argmin}\left\{-logp(y/\alpha) - logp(\alpha)\right\} \tag{3}$$

Where $p(y/\alpha) = \frac{1}{\sigma^n\sqrt{2\Pi}} exp(-\frac{1}{2\sigma_n^2}\|y - HD\alpha\|_2^2$.

The prior distribution of α is often characterized by an independent and identically distributed zero-mean Laplacian probability model is given by Equation 4,

$$p(\alpha) = \Pi_1^N \Pi_1^n \frac{1}{\sqrt{2}\sigma} exp(-\frac{1}{\sigma}\sqrt{2}\,|\,\alpha_{i,j}\,|) \tag{4}$$

Where σ is the standard deviation of the Laplacian model

The whole image can be sparsely represented by a set of coefficient vectors and the locally associated adaptive dictionaries. A collection of similar patches are grouped by PCA and PCA transformation matrix is computed over a set of training data. Since local patch can be well approximated by principal components the PCA transformation matrix naturally defines a dictionary for the given image structure. All image patches whose Euclidean distance to sparse codes is smaller than a threshold are selected as similar patches. By concatenating similar patches together, we obtain a 2D data matrix for sparse codes where each column is a similar patch vector extracted from the given image. Sparse codes are set and local dictionaries are initialized along with initial estimate sparse code of the given image. For the given set of vectors, PCA transformation coefficients are calculated with respect to threshold value. Dictionaries are updated iteratively to obtain the better results for the given input.

3.3. Locally and Non-Locally Centralized Sparse Representation

To reconstruct the degraded image, the sparse coding coefficients should be as close as possible to those of those of the unknown original image with the given dictionary. If only the local sparsity of the image is considered, the sparse coding co-efficient are often not accurate. To make the sparse coding more accurate, both the local and nonlocal sparsity constraint is considered. In Centralized Sparse Representation (CSR) modeling [6], Sparse Coding Noise (SCN) $\upsilon_{\alpha=}\alpha_{y-}\alpha_x$ is added to the original image. The sparse coding of x is based on y is given by Equations 5 and 6.

$$\alpha_y = \arg\min_{\alpha}\left\{\|y - H\phi o\alpha\|_2^2 + \lambda\|\alpha\|_1\right\} \tag{5}$$

$$\alpha_x = \arg\min_{\alpha}\left\{\|x - \phi o\alpha\|_2^2 + \lambda\|\alpha\|_1\right\} \tag{6}$$

The original image is first blurred by a Gaussian blur kernel with standard deviation 1.6 and Gaussian white noise of standard deviation √2 is added to get a noisy and blurred image. Each patch is individually coded and nonlocal similar patches to the given patch are clustered using PCA dictionary. Iteratively PCA dictionary can be used to code the patches for each cluster and dictionaries are updated along with the regularization parameters. The CSR model can be given by Equation 7.

$$\alpha_y = \underset{\alpha}{argmin} = \left\{ \| y - H\phi o\alpha \|_2^2 + \lambda \sum_i \| \alpha_i \|_1 + \gamma \sum_i \| \alpha_i - \beta_i \|_p \right\} \quad (7)$$

Where γ is a constant and l_p norm is used to measure the distance between α_i and β_i.

To improve the performance of IR, sparse coding noise is introduced in the CSR model. In CSR model only local sparsity is measured but in Nonlocally Centralized Sparse Representation (NCSR) model [9], image nonlocal self-similarity is exploited to obtain good estimates of the sparse coding co-efficient of the original image and then centralize the sparse coding coefficient of the practical image to those estimates. In CSR model, training patches are extracted from a set of images are clustered and PCA sub-dictionary for learning each cluster. In NCSR, adaptive sparse domain selection strategy is adopted to learn sub-dictionaries from the given image. Image patches are clustered by using the K-means clustering method K-PCA sub-dictionary construct a large over-complete dictionary to characterize all the possible local structures of natural images. In NCSR there is only one regularization term called NCSR and is given by Equation 8.

$$\alpha_y = \underset{\alpha}{argmin} \left\{ \| y - H\phi o\alpha \|_2^2 + \lambda \sum_i \| \alpha_i - \beta_i \|_p \right\} \quad (8)$$

Where $\|\alpha_i - \beta_i\|_p$ is the regularization term.

If the observation is maximum a posterior of α, we use argmax and also we apply zero mean Laplacian probability; the change in value will be minimum. So, the change in minimum value for sparse representation can be retained with help of all the above specified equations.

3.4. Low-Rank Approximation Structured Sparse Coding (LASSC)

CSR model utilize the nonlocal redundancies, leading to state-of-the-art image de-blurring results. In CSR model each patch is coded individually for the PCA dictionary. Instead of coding each patch individually, simultaneous sparse coding techniques code a set of patches simultaneously for the sparse code alignment [7]. Since patches share similar edge structures, over complete dictionary is not needed, a compact dictionary PCA. In image blurring using the patch based structured sparse coding model, structured sparsity over the grouped nonlocal similar patches can be enforced, patch clustering is updated for iterations.

4. Experimental Results

The selection of the de-noising technique is application dependent. So, it is necessary to learn and compare de-noising techniques to select the technique that is apt for the application in which we are interested. By far there is no criterion of image quality evaluation that can be accepted generally by all. A technique to calculate the signal to noise ratio in images has been proposed which can be used with some approximation. This method assumes that the discontinuities in an image are only due to noise. For this reason, all the experiments are done on an image with very little variation in intensity. The following Table 1 shows the Signal-to-Noise Ratio (SNR) values of the input and output images for the filtering approach.

Table 1. SNR results with Gaussian noise and standard deviation σ= 0.05.

Method	SNR Value Of Input Image	SNR Value Of An Output Image
Mean Filter	13.39	21.24
LMS Adaptive Filter	13.39	22..40
Median Filter	13.39	22.79

The following Figure 1 shows the result images from the mean filter, LMS adaptive filter and median filter when the Gaussian noise is added to the original image.

a) Original image with noise.

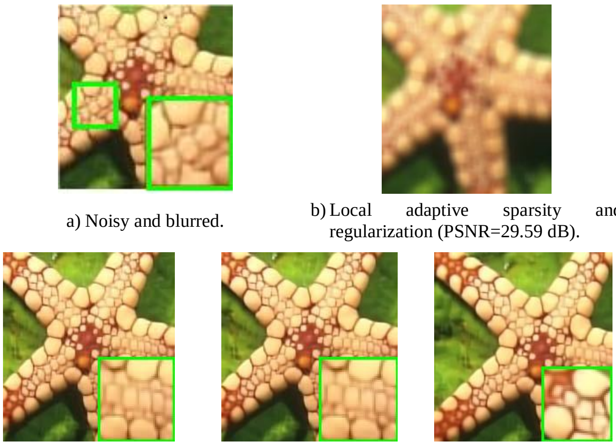
b) Result image using mean filter approach.

c) Result image using LMS adaptive approach.

d) Result image using median Filter.

Figure 1. De-noising performance comparison for the photograph image with standard deviation of σ=0.05 when Gaussian noise is added.

To verify the de-blurring performance, the experimental results of four different methodologies such as local adaptive sparsity and Regularization, CSR, LASSC, NCSR are shown in Figure 2. In all the four different methods, blurred images are obtained by adding the blur kernel and the additive Gaussian noise with standard deviation σ = √2 to the original test images two blur kernels, i.e., 9x9 uniform blur kernels and Gaussian blur kernel with standard deviation 1.6 are used for the simulation. The patches are extracted of the size 6x6 in case of LASSC and CSR, 7x7 in NCSR and 10x10 used in LASR. In NCSR non-blind de-blurring are conducted to further test the de-blurring performance under different blurring conditions. For real motioned blur images, blur kernel are borrowed from the kernel estimation method. The PSNR values are summarized in the Tables 2 and 3.



a) Noisy and blurred. b) Local adaptive sparsity and regularization (PSNR=29.59 dB).

c) CSR (PSNR=30.30 dB). d) LASSC (PSNR=30.72). e) NCSR (PSNR=30.28).

Figure 2. De-blurring performance comparison for starfish image (9x9 uniform blur, $\sigma_n=\sqrt{2}$).

Table 2. PSNR results (9x9 uniform blur, $\sigma_{n=\sqrt{2}}$).

Image	Butterfly	Parrot	Starfish	Barbara	Leaves
LASR	28.63	31.22	29.66	27.34	28.51
CSR	29.75	32.09	30.30	27.93	29.97
NCSR	29.68	31.95	30.28	28.10	29.98
LASSC	30.16	32.33	30.72	28.40	30.65

Table 3. PSNR results (Gaussian blur, $\sigma_{n=\sqrt{2}}$).

Image	Butterfly	Parrot	Starfish	Barbara	Leaves
LASR	29.72	32.33	31.51	27.37	30.41
CSR	30.79	33.41	32.29	28.44	31.74
NCSR	30.75	33.44	32.31	27.81	31.44
LASSC	30.84	33.39	32.27	27.91	31.57

From the result it is shown that NCSR model performs better than other methods namely, CSR, LASR and LASSC. All restoration algorithms subjected to study in this paper, irrespective of the type of blur kernel, performs badly on the gray scale image. The average PSNR of all the algorithms on the test images taken, explains that NCSR performs better than LASR and on par with CSR. But LASSC seems to perform better than NCSR only marginally, in case of uniform blur. In case of Gaussian blur also, the same inference can be drawn. Comparison of PSNR obtained for all the five test images explains that they all perform better on a low contrast image like parrot, than on a high contrast image like leaves and butterfly, irrespective of the blur kernel. It is also seen from the individual and average PSNR values that, images blurred with Gaussian blur kernel exhibits better PSNR than that blurred with 9X9 uniform blur kernel. Observing the worst case (leaves) and the best case (parrot) in color images for both types of blur kernel, it is understood that NCSR is better than CSR and LASR. LASSC performs better than NCSR only by a very marginal value. The NCSR approach achieves higher performance for the image de-blurring, de-noising and super resolution methods.

5. Conclusions

In this review paper, a comparative study is presented for three different de-noising techniques of image restoration. With the original image, an additive Gaussian noise is added with standard deviation 0.05. The experimental result shows that median filter performs better when Gaussian noise is added than other de-noising techniques such as mean filter and LMS adaptive filter. When we are analyzing the sparse representation, the above three techniques analyzed and evaluated are used to remove the noise from the images as well as de-blurring the original images.

Along with the de-noising techniques, we present a comparative study for four different types of image restoration methods with respect to sparse representation. Experimental results shows that LASSC performs better when blur kernel is added. NCSR has a better PSNR value with respect to Gaussian kernel. We would like to work on these aspects along with multimodal sparse coding on IR as part of our future work.

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