

# Evaluation for Diaphragm's Deflection for Touch Mode MEMS Pressure Sensors

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**Abstract:** In this paper, an analytical and simulation solution for touch mode Micro-electromechanical systems pressure sensor operating in harsh environment is proposed. The principle of the paper is to design, obtain analytical solution and compare the results with the simulation using finite elements analysis for a circular diaphragm deflection before and after touch point. By looking at MEMS devices, when the diaphragm starts touching the fixed electrode by applying loads, it will have a major effect on the overall performance of the device. Therefore, one should consider the effect of touch mode in the system to achieve good linearity, large operating pressure range and large overload protection at output. As of so far the effect of touch mode has not been evaluated efficiently in the literatures. The proposed touch mode MEMS capacitive pressure sensor demonstrated diaphragm with radius of 180  $\mu\text{m}$ , the gap depth of 0.5  $\mu\text{m}$  and the sensor exhibit a linear response with pressure from 0.05 Mpa to 2 Mpa.

**Keywords:** MEMS, Touch mode, capacitive pressure sensor, harsh environment, FEA, and circular diaphragm.

Received October 27, 2008; accepted May 17, 2009 Received

## 1. Introduction

A simulation solution is one of the valuable processing for design a sensor. A circular diaphragm with clamped edges and a constant residual stress due to using same materials for diaphragm and substrate was modeled in Finite Elements Analysis (FEA). One of the main mechanisms behind the variation of the capacitive pressure sensor is to evaluate capacitance between two electrodes and contact area. So far, simulation based on FEA are widely used to model touch mode pressure sensors, but it is very time consuming to optimize the radius, thickness of the diaphragm and cavity depth between two electrodes. The proposed capacitive pressure sensor using silicon carbide as of material for harsh environments, silicon carbide owing excellent electrical stability, mechanical robustness and chemical inertness properties [5], low turn-on temperature drift, having high sensitivity, wireless sensing schemes and a minimum dependence on side stress [1, 8].

## 2. Design Process

High temperature pressure sensors are critical for advanced industrial, automotive, aerospace, gas turbine, oil/logging equipments, nuclear station, and power station applications [5]. Due to limitation exist for high temperature silicon's material properties; this device is not adequate to be use for designing MEMS

sensor in harsh environment (high temperature). In this paper it demonstrated the simulation of the MEMS capacitive pressure sensor focused on touch mode to show good linearity, large operating pressure range and large overload protection at output. Figures 1 and 2 present a cross-sectional view of a touch mode and normal mode operation of MEMS capacitive pressure sensor. In touch mode operation, when external pressure increases, the diaphragm will deflect toward inside and the diaphragm start touching the bottom electrode with a distance of insulator in between. In normal mode operation, the diaphragm is kept distance away from bottom electrode [4]. The sensor consists of two parallel circular plates with clamped-edges, suspended over a sealed cavity. The concept of parallel plate capacitor is expressed by equation 1.

$$C = \epsilon_0 \epsilon_r \frac{A}{d} \quad (1)$$

where  $\epsilon_0$  is the permittivity of the media between plates is,  $\epsilon_r$  is the dielectric constant of the material between the plates of the capacitance,  $A$  is the area of the electrode, and  $d$  is the gap between two plates.

The concept of the capacitance element of the sensor requires a change in the capacitance as a function of some applied pressure load. A realization function of this concept would be the plates of the capacitor could move under pressure load, for example if the plates move closer together, the gap height,  $g$ , would decrease, resulting an increase in

capacitance of the sensor. In touch mode, when external pressure increases on the diaphragm, the touched radius ( $r_1$ ) will increase, and at the same time the untouched radius ( $r_2$ ) will decrease, therefore the value of capacitance will increase nearly linearly with increasing pressure, before touch point the radius ( $r_L$ ) is zero. As shown in Figure 1  $r$ ,  $r_1$ ,  $r_2$  are defined radial distance from centre, touched-point radius, and untouched-point radius respectively.  $t_1$ ,  $t_2$  are defined the thickness of dielectrics respectively.  $g$  is defined cavity depth,  $h$  is the thickness of the diaphragm [3].

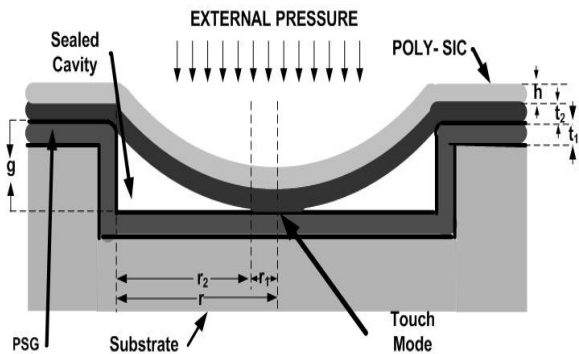


Figure 1. Cross-sectional view of touch mode pressure sensor.

### 3. Theory of Operation

A plate defined thin plate or small deflection if the gap between two electrodes is less than 1/5 of diaphragm's thickness, and the strains and mid-plane slopes are much smaller than unity. A plate defined as of thick plate or large deflection if its deflection is up three times larger than diaphragm's thickness [10]. Based on small deflection theory for circular plate, the deflection  $w$  of any point on a circular plate under uniform pressure is expressed by the following partial equation (eFunds).

$$\nabla^2 w \nabla^2 D = P \tag{2}$$

where  $P$  applied pressure (force per unit area) acting in the same direction as  $Z$ ,  $D$  is the flexural rigidity of the plate is given by Figure 2.

$$D = \frac{Eh^3}{12(1-\nu^2)} \tag{3}$$

where  $E$ , is Young's Modulus,  $h$  is the thickness of diaphragm,  $\nu$  is defined the Poison's ratio. The differential operator  $\square_2$  is called the Laplacian differential operator. For circular plate is simply supported classical formula and is defined by

$$\Delta \equiv \nabla^2 = \frac{\partial^2}{\partial r^2} + \frac{1}{r^2} \frac{\partial^2}{\partial \phi^2} + \frac{1}{r} \frac{\partial}{\partial r} \tag{4}$$

if the bending rigidity  $D$  is constant through-out the plate, the deflection equation 2 for Cylindrical

coordinate (circular plates) can be simplify and given by

$$\nabla^4 w = \frac{P}{D} \tag{5}$$

where, equation 5 is called the bi-harmonic differential operator. The deflection for any point of the plate at ( $0 < r < a$ ) would be

$$w(r) = \frac{Pa^4}{64D} \left( 1 - \left( \frac{r}{a} \right)^2 \right)^2 \tag{6}$$

where,  $a$  is the radius of the plate, and  $r$  is the radial distance from the center, the maximum deflection of the plate is located in the center point of the plate ( $r = 0$ ), and defined by

$$w_0 = \frac{Pa^4}{64D} \tag{7}$$

Figure 2, shows the deflection  $w(r)$  before touch of the circular plate as a function of radius is given by equation 6 and maximum center deflection  $w_0$  is defined by equation 7. Equation 6 is valid before the diaphragm touches the bottom electrode (with the insulator in between the plate), then the shape of the bending line deviates extremely from expression 6. For every pressure, the radius of the diaphragm that barely contacts the bottom is called untouched radius ( $r_2$ ), and touched radius ( $r_1$ ) will be calculated by subtracting the total radius  $a$  from  $r_2$ , noted that the touched radius was zero before touch, as shown in Figure 1.

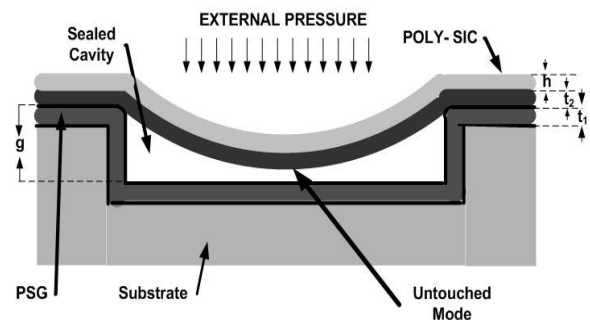


Figure 2. Cross-sectional view of normal mode pressure sensor.

### 4. Simulation Modeling and Analytical Analysis Before and After Touch-Mode Effect

In the modeling of MEMS, touch mode effect is the most difficult subject due to deflecting of the top diaphragm, and the movement of top diaphragm should be considered whenever it touches the bottom electrode with an insulator in between. There have been good suggestions in the literatures for solving the touch-down problems, but as of today none of the literatures has come up to a final analytical solution for calculating the touch-down effect [6, 7]. Another alternative to solve the existing problem by considering to changing the boundary conditions by writing a power series equation for diaphragm

deflection before and after touch-down. To obtain a solution of a uniformly loaded circular plate with a clamped edge, it is necessary to consider the equilibrium conditions of the element of circular plate. To achieve that, one first has to write the equations in a different form. In summary, we have a set of governing equations as follows [9]:

$$N_r + r \frac{dN_r}{dr} - N_t = 0 \quad (8)$$

$$D \left( \frac{d^3 w}{dr^3} + \frac{1}{r} \frac{d^2 w}{dr^2} - \frac{1}{r^2} \frac{dw}{dr} \right) = N_r \frac{dw}{dr} + \frac{qr}{2} \quad (9)$$

$$r \frac{d}{dr} (N_r + N_t) + \frac{hE}{2} \left( \frac{dw}{dr} \right)^2 = 0 \quad (10)$$

For solving power series, solution of circular diaphragm under uniform load with clamped edge, a set of non-uniform differential equations by transforming the equations to a dimensionless form  $(p, \gamma, S_r, S_t)$ , and introduced by the following notations:

$$p = \frac{q}{E}, \quad \gamma = \frac{r}{h}, \quad S_r = \frac{N_r}{Eh}, \quad S_t = \frac{N_t}{Eh} \quad (11)$$

with given dimensionless notations equation 11 the governing equations become as follows:

$$\frac{d}{d\gamma} (\gamma S_r) - S_t = 0 \quad (12)$$

$$\frac{1}{12(1-\nu^2)} \frac{d}{d\gamma} \left[ \frac{1}{\gamma} \frac{d}{d\gamma} (\gamma \frac{dw}{dr}) \right] = \frac{p\gamma}{2} + S_r \frac{dw}{dr} \quad (13)$$

$$\gamma \frac{d}{d\gamma} (S_r + S_t) + \frac{1}{2} \left( \frac{dw}{dr} \right)^2 = 0 \quad (14)$$

The boundary conditions in this case require the radial displacement and the slope  $\frac{dw}{dr}$  vanish at the boundary. The solution can be represented the function by the following power series [9]:

$$S_r = B_0 + B_2 \gamma^2 + B_4 \gamma^4 + \dots \quad (15)$$

Substituting in equation 14 and solve for  $S_t$ , is given by:

$$S_t = B_0 + 3B_2 \gamma^2 + 5B_4 \gamma^4 + \dots \quad (16)$$

$$\left( \frac{dw}{dr} \right) = \sqrt{8} (C_1 \gamma + C_3 \gamma^3 + C_5 \gamma^5 + \dots) \quad (17)$$

By integration equation 17, we find

$$\frac{w}{h} = \sqrt{8} \left( C_0 + C_1 \frac{\gamma^2}{2} + C_3 \frac{\gamma^4}{4} + C_5 \frac{\gamma^6}{6} + \dots \right) \quad (18)$$

All of the equations must be satisfied for any value of  $\gamma$ , and find the relations between the constants  $B$  and  $C$ :

$$B_k = -\frac{4}{k(k+2)} \sum_{m=1,3,5,\dots}^{k-1} C_m C_{k-m} \quad k=2,4,6,\dots \quad (19)$$

$$C_k = \frac{12(1-\nu^2)}{k^2-1} \sum_{m=0,2,4,\dots}^{k-3} B_m C_{k-2-m} \quad k=5,7,9,\dots \quad (20)$$

$$C_3 = \frac{3}{2} (1-\nu^2) \left( \frac{p}{2\sqrt{8}} + B_0 C_1 \right) \quad (21)$$

where  $B_0, B_2, \dots$ , and  $C_0, C_1, C_3, \dots$ , are constants to be determine, where two constants  $B_0$  and  $C_1$  are assigned by iteration, all other constants are determined by substituting in given equations for different boundary conditions. In order to include the touch down effect into a behavioral model, we should calculate the radius  $r_1$  (touched radius) that is a function of touched-mode pressure  $P_T$ , for calculation of bending line in touched-mode, the power series equations (19-21) modified by substituting  $\gamma$  by  $\frac{r-r_1}{h}$ , and is given by the following [10]:

$$S_r = B_0 + B_2 \left( \frac{r-r_1}{h} \right)^2 + B_4 \left( \frac{r-r_1}{h} \right)^4 + \dots \quad (22)$$

$$S_t = B_0 + 3B_2 \left( \frac{r-r_1}{h} \right)^2 + 5B_4 \left( \frac{r-r_1}{h} \right)^4 + \dots \quad (23)$$

$$\left( \frac{dw}{dr} \right) = \sqrt{8} \left( C_1 \left( \frac{r-r_1}{h} \right) + C_3 \left( \frac{r-r_1}{h} \right)^3 + C_5 \left( \frac{r-r_1}{h} \right)^5 + \dots \right) \quad (24)$$

$$\frac{w}{h} = \sqrt{8} \left( C_0 + C_1 \frac{(r-r_1)^2}{2h^2} + C_3 \frac{(r-r_1)^4}{4h^4} + C_5 \frac{(r-r_1)^6}{6h^6} + \dots \right) \quad (25)$$

It is not practical to determine four unknown constants  $B_0, C_0, C_1$  and  $r_1$  by assigning value. As we know  $r_1$  before touch is equal to zero and touch point pressure ( $P_T$ ) is given when the load pressure is more than touch point pressure. [10]:

$$P_T = \frac{64 D w_0}{r^4} \quad (26)$$

Now, calculate the equivalent untouched radius ( $r_2$ ) of a virtual circular diaphragm by using equation 28, and is defined by:  $w = g_0, 2r = r$

$$r_2 = \sqrt[4]{64 D g_0 / P_T} \quad (27)$$

Touched- point radius  $r_1$  is given by:

$$r_1 = a - r_2 \quad (28)$$

$C_0$  Can be defining by the maximum deflection equation 8 of circular diaphragm before and after touch and is given by, before touch point:

$$C_0 = \frac{w_0}{\sqrt{8} h} = \frac{1}{\sqrt{8} h} \frac{P_T a^4}{64 D} \quad (29)$$

After touch point where  $w_0 = g$ , by satisfying the boundary condition at the touch radius and is given by:

$$w|_{r=r_1} = g, \quad \left. \frac{dw}{dr} \right|_{r=a} = 0, \quad C_0 = \frac{w_0}{\sqrt{8h}} = \frac{g}{\sqrt{8h}} \quad (30)$$

Hence, the value of  $r_1$  touched radius and  $C_0$  constant have been determined, the other two constants  $B_0$  and  $C_1$  can be assigned proper value to satisfy the boundary conditions at  $r = a$  equation 6 and is given by:

$$w|_{r=a} = 0, \quad \left. \frac{dw}{dr} \right|_{r=a} = 0$$

Finally by having these constants, we can find a set of  $B_k$  and  $C_k$ .

### 5. Simulation Results

Figure 3 shows the comparison result of the Analytical and FEM of the radial distance versus diaphragm deflection at different pressure loads. The test model is designed, with 180 μm radius, 4 μm diaphragm thickness, 0.5 μm of cavity depth, and operates with the pressure up to 2Mpa. Figure 3(a) shows the deflection with the applied pressure from 10kpa-55kpa before touch point and Figure 3(b) shows the deflection with the applied pressure from 100kpa-500kpa after touch point.

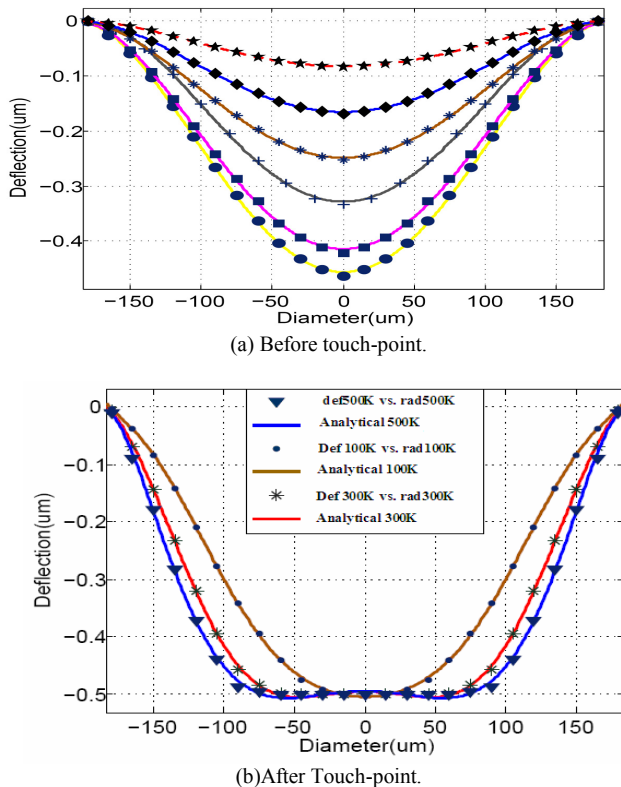


Figure 3. Radial distance (diameter) vs. deflection.

Figures 4 and 5 show the 3D-Visualize pressure load. Theoretical model for evaluating the change of capacitance after touch mode in Table 1 shows FEM results pressure loads vs. touched radius and pressure vs. capacitance at different pressure range. The model is designed from a=180 μm radius, h= 4 μm diaphragm thickness, g= 0.5 μm Cavity depth.

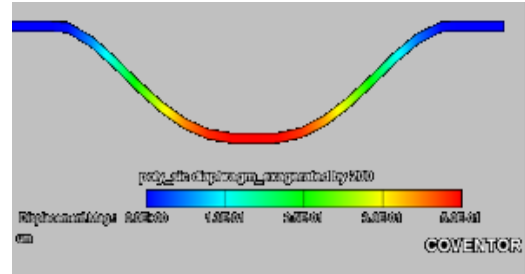
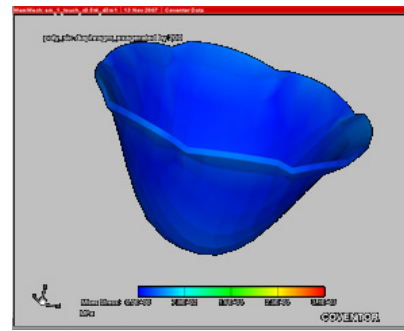
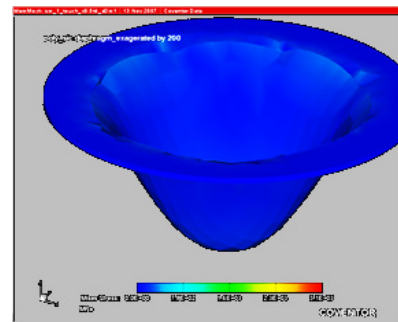


Figure 4. 3D\_visualize side view touch-mode contact point\_0.1Mpa.



(a) With no clamp.



(b) With clamp.

Figure 5. 3D-visualize Pressure vs. deflection with clamp.

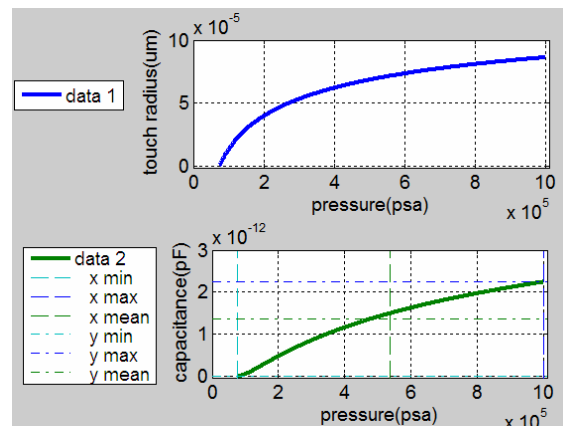


Table 1. Pressure range from 75Kpa to 1Mpa vs.

Statistic	Pressure(Psa)	Touch Radius
Min.	75000	7.1985e-7
Max.	1000000	8.618e-5
Mean	5.375e5	6.4123e-5
Median	5.375e5	7.0427e-5
Std.	2.6746e5	1.9997e-5
Range	9.25e5	8.546e-5

Statistic	Pressure(Psa)	Capacitance(pF)
Min.	75000	1.5642e-16
Max.	1000000	2.2419e-12
Mean	5.375e5	1.3617e-12
Median	5.375e5	1.4972e-12
Std.	2.6746e5	6.4929e-13
Range	9.25e5	2.2417e-12

## 6. Conclusions

The results for analytical and FEA is presented to evaluate before and after touch mode circular diaphragm at different applied pressure loads. These methods are widely used to model MEMS pressure sensors, but simulating in FEA is time consuming to optimize sensor's parameters such as: radius, cavity depth, diaphragm and dielectric thickness, Young's modulus, Thermal Coefficient Expansion (TCE) and etc. the solution involved deflection and power series theories for before and after touch-point using touch mode capacitive pressure sensor, have the advantage of good linearity, it has shown exact contact deformation, pressure vs. deflection, pressure vs. capacitive proposed by Timoshenko's theories and also by Wen H. Ko. The two results by using FEA and analytical was very promising results, especially for calculating the touch-point radius approximation.

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