Representing Uncertainty in Medical Knowledge: An Interval Based Approach for Binary Fuzzy Relations

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Abstract: *This paper addresses issues involved in representation of causal relationships between medical categories. An interval based approach for medical binary fuzzy relations is proposed to represent the ignorance about uncertainty and imprecision. A major advancement propagated by this model lies in formalizing some novel medical measures enhancing the sight in understanding the causality relationship between medical entities. This view is expressed in extension of the classical fuzzy implication relationship in terms of interval valued fuzzy inclusion relationship in the context of fuzzy binary relationships. The focus of attention of this model is based on utilizing interval based fuzzy inclusion relationships as causality measures expressing the strength of the degree of inclusion between fuzzy sets. In addition, derived from the direction of an inclusion degree, an interval based causal relationship can medically be interpreted as the necessity or the sufficiency of occurrence of a medical entity such as symptoms or disease with another one. Furthermore, for simplification of computations and defuzzification of dependent intervals a method for transformation of these relations into point-valued relations is proposed.*

Keywords: *Binary fuzzy relation, interval valued representation, medical knowledge representation, fuzzy inclusion measure, uncertainty, fuzzy logic.*

Received April 23, 2008; accepted September 1, 2009

1. Introduction and Motivation

Establishing causal and associational relationships between clinical categories such as clinical observations; i.e., findings or symptoms, path physiological states, diagnosis or diseases, tests and therapies can be represented in different ways and at different levels. However, as most medical knowledge is uncertain and or imprecise, representing medical knowledge requires methods capable of dealing with uncertainty and imprecision such as numerical and symbolic methods. On the other hand, obtaining pure statistical precise values for representing uncertain or imprecise relationships might be very complex to acquire. Even quantitative medical information coming from measurement are never 100% accurate, they are usually expressed in terms of intervals considering the measurement error factor. Moreover, knowledge bases could possibly be affected with logical inconsistency or incompleteness.

This work is hence attempting to introduce a formal knowledge representation model discussing the prospect of utilizing the fuzzy inclusion relationship to represent significant medical measures between fuzzy sets representing medical entities. In this context, illdefined causal relationships between medical entities are captured through key medical measures such as the necessity and sufficiency of occurrence. These basic relationships can be interpreted as uncertain rules expressing the degree of impression and uncertainty between the antecedents and the consequents expressed by the degree of fuzzy inclusion. Furthermore, to handle the problem of ignorance about the uncertainty this model is proposing to extend such fuzzy Subsethood or inclusion relations to consistent interval-based relationships.

In this model, the direction of an inclusion relationship between fuzzy sets plays a decisive role in the interpretation of a medical causal relationship. Furthermore, an interval based causal relationship can medically be represented at different levels of the clinical knowledge, e.g., patient symptom manifestation, observational, path physiological, disease level, *etc*. An important enhancement lies in extension of the classical fuzzy implication relationship in terms of interval valued fuzzy inclusion relationship in the context of fuzzy binary medical relationships. Additionally, a transformation of these relations into point valued relationship is also proposed for reducing and simplification of the computability of such relationships.

1.1. Scope and Related Work

The importance of the concept "fuzzy inclusion" or "fuzzy subsethood", has already been stressed by many authors [7, 8, 11, 18]. Meanwhile, it has been applied to different areas of computing such as image processing, neural network architecture and in medicine. On the level of representation of medical knowledge, there is some comparable work to this approach in recognizing the significance of utilizing fuzzy inclusion relationships as a departure point in representing clinical medical knowledge [8]. In [10] the authors stressed on perception based reasoning using medical measures such as necessary causal ground and sufficient causal ground, which can be derived from the fuzzy cardinality and fuzzy subset hood. Also, the different implementations of CADIAG-II medical fuzzy expert system [3, 13] have a connection to the proposed model in the context of considering point valued binary fuzzy relations such as strength of conformation, frequency of occurrence and utilizing type-2 fuzzy sets in representing clinical medical knowledge with the significant difference, that the proposed model is proceeding from bidirectional interval valued binary fuzzy relations or rules which are based on consistent interval propagations [8]. On the general level of representation of uncertainty and ignorance, interval valued methods have been proposed by many different researchers. Zadeh [19] proposed type-2 fuzzy sets, whose membership functions themselves are characterized by fuzzy sets. Baldwin [2] employed necessity and possibility support boundaries assigned to horn clauses to represent the uncertainty. Turksen [17] proposed compositional operations based on the disjunctive, conjunctive, and normal forms for treating approximate analogical reasoning.

In context of modeling uncertain knowledge based on conditional probability as an uncertain implication [15], and others, there is some corresponding analogy to the proposed model in regarding conditional probabilities as a form of the classical subsethood relationships. Finally, the concept of fuzzy quantifiers based on cardinality of fuzzy sets [1, 4, 16] can be regarded as imprecise quantification over subsethood relationships. Many researchers proceed from analyzing properties of the conditional probabilities as a probabilistic measure and not from the fuzzy inclusion as a measure for measuring the causal and associative relationships between medical entities. There is also some work considering fuzzy Interval as an approach for knowledge representation [12].

In the following, we will discuss some aspects of utilizing interval based binary fuzzy relations characterized by membership functions assigning consistent intervals expressed in terms of consistent values for possible degrees for an inclusion relationship between two fuzzy sets. In the next sections, some preliminary definitions and some new measures will be introduced. These measures are concerned with interval based representations such as forward and backward inclusion relations between medical entities to describe important medical measures such as necessity and sufficiency measures and their medical representation. Subsequently, a point valued computational model will additionally be presented in the context of dependent and consistent medical knowledge bases.

2. Measures for Representing Medical Knowledge

In the following some preliminary and basic notion are introduced: a fuzzy set A on a universe $\mathbb{U}=\{x_1,x_2,\ldots,x_n\}$ is defined by a membership function $\mu_A : U \rightarrow [0,1]$ and $\mu_A(x)$ is the grade of membership of the element x in A. In this paper fuzzy sets are represented by capitals, e.g. A, B, …, universal sets by U, S, D, …

Definition 1: the cardinality of a fuzzy set A on U is defined as follows:

$$|A| = \sum_{x\in\mathbb{U}} \mu_A(x) \quad (1)$$

Generally, $|A|$ is a real number. This approach is proceeding form scalar cardinality of fuzzy sets and not from a fuzzy cardinality for a fuzzy set [5, 6, 14]. A major advantage of adopting this definition lies in the fact that some basic properties hold in the scalar cardinality in a simplified form such as:

- Monotonicity; i.e., $A \subseteq B \Rightarrow |A| \leq |B|$
- Coverage property; i.e., $|\neg A| = |\mathbb{U}| - |A|$
- Additivity rule; i.e., $|A \cup B| + |A \cap B| = |A| + |B|$

These properties are relevant to the proposed model.

2.1. Measures for Medical Binary Relationships

In the following, some novel uncertain measures are introduced:

Definition 2: based on [8, 11], the degree of inclusion of a fuzzy set in another one is defined by the fuzzy subsethood. For the fuzzy sets A and B on U, the degree of a forward inclusion $\overrightarrow{FI}(A,B)$ is defined as follows:

$$\overrightarrow{FI}(A,B) \triangleq \begin{cases} \dfrac{\sum_{x\in\mathbb{U}} Min(\mu_A(x),\mu_B(x))}{\sum_{x\in\mathbb{U}} \mu_A(x)} \in [0,1] & A \neq 0 \\ 1 & A = 0 \end{cases} \quad (2)$$

Analogue, a backward inclusion $\overleftarrow{BI}(A,B)$ is defined as the fuzzy inclusion degree to which B is contained in the fuzzy set A:

$$\overleftarrow{BI}(A,B) \triangleq \begin{cases} \dfrac{\sum_{x\in\mathbb{U}} Min(\mu_A(x),\mu_B(x))}{\sum_{x\in\mathbb{U}} \mu_B(x)} \in [0,1] & B \neq 0 \\ 1 & B = 0 \end{cases} \quad (3)$$

fuzzy inclusion relationships expresses the degree of inclusion of a fuzzy set in another one. For example

$A \xrightarrow[\overrightarrow{FI}(A,B)]{} B$ expresses the degree to which the fuzzy set A is a subset of the fuzzy set B, which can be interpreted as a forward uncertain rule [8].

Definition 3: let $\mathbb{S} = \{ S_1, S_2, ..., S_n \}$ be a universal set representing the clinical category findings or symptoms and $\mathbb{D} = \{ D_1, D_2, ..., D_m \}$ a universal set representing the clinical category pathophysiological states or diseases, then an interval valued necessary causal measure, $\overleftarrow{R}_{BSD}$, is defined as a backward interval valued binary fuzzy inclusion relation on the Cartesian product $\mathbb{S} \times \mathbb{D}$, where

$$\overleftarrow{R}_{BSD} \triangleq \{((S_i, D_j), \mu_{\overleftarrow{R}_{BSD}}(S_i, D_j)) \mid (S_i, D_j) \in \mathbb{S} \times \mathbb{D}\} \quad (4)$$

and

$$\mu_{\overleftarrow{R}_{BSD}}(S_i, D_j) \triangleq [\overleftarrow{BI}(S_i, D_j)_{min}, \overleftarrow{BI}(S_i, D_j)_{max}] \subseteq [0,1] \quad (5)$$

where

$$\overleftarrow{BI}(S_i, D_j) \in [\overleftarrow{BI}(S_i, D_j)_{min}, \overleftarrow{BI}(S_i, D_j)_{max}] \quad (6)$$

The direction of $\overleftarrow{R}_{BSD}$ is of medical importance. $\overleftarrow{R}_{BSD}$ can medically be interpreted as a relation expressing the necessity of occurrence in terms of an interval. Considering $\overleftarrow{BI}(S_i, D_j)$ as a truth-value for a fuzzy rule such as $S_i \xleftarrow[[\overleftarrow{n}_{min}, \overleftarrow{n}_{max}]]{} D_j$ expresses, that the boundaries for the necessity of occurrence for an implication relation should lie within the interval $[\overleftarrow{n}_{min}, \overleftarrow{n}_{max}]$. Thus, if S_i is completely necessary for D_j; i.e., $S_i \xleftarrow[1]{} D_j$, then there is no antecedent D_j without occurring the consequent S_i. In this case S_i is obligatory or occurs always with D_j. In the case of $S_i \xleftarrow[[0.5, 0.75]]{} D_j$, S_i is necessary for D_j to the grade lying within [0.5, 0.75], i.e., the degree to which the antecedent S_i is present whenever the consequent D_j is present should be in [0.5, 0.75].

The intervals are supposed to be confident and minimal; i.e., the actual degree of the necessity is guaranteed to be present within a minimal interval $\mu_{\overleftarrow{R}_{BSD}}(S_i, D_j)$ according to an incrementally constructed consistent knowledge base. Details for checking the consistency of such knowledge are found in [8].

As the amount $|D_j| - |S_i \cap D_j|$; i.e., the amount of D_j outside of $S_i \cap D_j$ weakens the causal relation between S_i and D_j, the square of the inclusion degree $\overleftarrow{BI}(S_i, D_j)$ might be used as necessary causal ground as proposed in [10]:

$$\text{degree}(S_i \subset D_j)^2 \triangleq \left(\frac{|S_i \cap D_j|}{|S_i|} \right)^2 \quad (7)$$

The necessary causal ground is used as a clinical measure for measuring the sensitivity of an individual patient or object of interest to an initial condition; i.e., the fuzzy set A, which might be changed after change or therapeutic intervention, represented by B state. In this work, the necessary inclusion relation as a causal relation will be used as defined in definition 3.

Definition 4: $\overrightarrow{R}_{FSD}$ is defined as a forward interval valued binary fuzzy inclusion relation on the Cartesian product $\mathbb{S} \times \mathbb{D}$:

$$\overrightarrow{R}_{FSD} \triangleq \{((S_i, D_j), \mu_{\overrightarrow{R}_{FSD}}(S_i, D_j)) \mid (S_i, D_j) \in \mathbb{S} \times \mathbb{D}\} \quad (8)$$

and

$$\mu_{\overrightarrow{R}_{FSD}}(S_i, D_j) \triangleq [\overrightarrow{FI}(S_i, D_j)_{min}, \overrightarrow{FI}(S_i, D_j)_{max}] \subseteq [0,1] \quad (9)$$

where

$$\overrightarrow{FI}(S_i, D_j) \in [\overrightarrow{FI}(S_i, D_j)_{min}, \overrightarrow{FI}(S_i, D_j)_{max}] \quad (10)$$

$\overrightarrow{R}_{FI}$ in definition 2 , in the context of causality can be interpreted as a relation expressing the sufficiency of occurrence in terms of an interval. Fuzzy rules having the measure $\mu_{\overrightarrow{R}_{FSD}}(S_i, D_j)$ as a truth-value such as $S_i \xrightarrow[\mu_{\overrightarrow{R}}(S_i, D_j)]{} D_j$ should be minimal, guaranteed and consistent. Thus, if S_i is completely sufficient for D_j; e.g., $S_i \xrightarrow[[1, 1]]{} D_j$ then there is no S_i without occurring D_j. That means, the antecedent S_i is sufficient for the consequent D_J. In the case of $S_i \xrightarrow[[0.75, 0.9]]{} D_j$, S_i is sufficient for D_J to the grade which has to lie in [0.75, 0.9], i.e., the degree to which D_J is present whenever S_i is present.

Definition 5: a bidirectional interval-valued relation $\overleftrightarrow{R}_{FB}$ is defined as simultaneous existence of the $\overrightarrow{R}_{FSD}$ and $\overleftarrow{R}_{BSD}$:

$$\overleftrightarrow{R}_{FB} \triangleq \{(((S_i, D_j), \mu_{\overrightarrow{R}_{FSD}}(S_i, D_j)),((S_i, D_j), \mu_{\overleftarrow{R}_{BSD}}(S_i, D_j))) \mid (S_i, D_j) \in \mathbb{S} \times \mathbb{D}\} \quad (11)$$

The Compound Causal Measure (CCM), is defined as the minimum of $\mu_{\overrightarrow{R}_{FSD}}(S_i, D_j)$ and $\mu_{\overleftarrow{R}_{BSD}}(S_i, D_j)$

$$\begin{aligned} &[\overrightarrow{FI}(S_i, D_j)_{min}, \overrightarrow{FI}(S_i, D_j)_{max}] \wedge \\ &[\overleftarrow{BI}(S_i, D_j)_{min}, \overleftarrow{BI}(S_i, D_j)_{max}] \triangleq \\ &[MIN(\overrightarrow{FI}(S_i, D_j)_{min}, \overleftarrow{BI}(S_i, D_j)_{min}), \\ &MIN(\overrightarrow{FI}(S_i, D_j)_{max}, \overleftarrow{BI}(S_i, D_j)_{max})] \end{aligned} \quad (12)$$

Bidirectional causal measure expresses the degree to which both necessity and sufficiency are simultaneously present. In the maximal case when observing all patients having $|S_i \cap D_j|$, S_i is sufficient and necessary for D_j e.g., $S_i \underset{1}{\overset{1}{\rightleftarrows}} D_j$, i.e., S_i=D_j. In general, the more S_i and D_J are distinct, the less the causal flow from S_i to D_j and the more something else besides S_i is causing D_j. The meaning of compound measure might be explained in the difference of the initial state of a patient before changes and intervention or therapies [10], that separates the patient from the disease D_j. A patient, who does not exhibit any signs of

S_i should be separated from D_j, e.g., after the changes caused by the therapy**.** The negation of the compound causal measure should therefore express the difference between S_i and D_j as fuzzy sets. In the case of $S_i \xrightleftharpoons[{[0.5,0.6]}]{[0.75,0.9]} D_j$, $S_i \xleftrightarrow[{[0.5,0.6]}]{} D_j$ represents the compound interval valued causal measure, i.e., S_i is necessary and sufficient to D_j to the degree lying in [0.5, 0.7]. Furthermore, in this context, point valued causal relationships can be considered as a special case of the compound interval based measure:

Definition 6: $\overleftrightarrow{R}_{FBp}$ is defined as simultaneous existence of the point-valued $\overrightarrow{R}_{FSDp}$ and $\overleftarrow{R}_{BSDp}$ as shown in definitions 2 and 3:

$$\overleftrightarrow{R}_{FB_p} \triangleq \{(((S_i, D_j), \mu_{\overrightarrow{R}_{FSDp}}(S_i, D_j)),((S_i, D_j), \mu_{\overleftarrow{R}_{BSDp}}(S_i, D_j))) \mid (S_i, D_j) \in \mathbb{S} \times \mathbb{D}\} \tag{13}$$

$\overleftarrow{R}_{BSDp}$ is defined as a backward point-valued binary fuzzy inclusion relation on the Cartesian product $\mathbb{S} \times \mathbb{D}$, where

$$\overleftarrow{R}_{BSD_p} \triangleq \{((S_i, D_j), \mu_{\overleftarrow{R}_{BSDp}}(S_i, D_j)) | (S_i, D_j) \in \mathbb{S} \times \mathbb{D}\} \tag{14}$$

and

$$\mu_{\overleftarrow{R}_{BSDp}}(S_i, D_j) \triangleq \overleftarrow{B}I(S_i, D_j) \in [0,1] \quad , \tag{15}$$

$$\overrightarrow{R}_{FSD_p} \triangleq \{((S_i, D_j), \mu_{\overrightarrow{R}_{FSDp}}(S_i, D_j)) | (S_i, D_j) \in \mathbb{S} \times \mathbb{D}\} \tag{16}$$

where

$$\mu_{\overrightarrow{R}_{FSDp}}(S_i, D_j) \triangleq \overrightarrow{F}I(S_i, D_j) \in [0,1] \tag{17}$$

3. A Framework for Representation Binary Medical Causal Relationship

Considering medical entities as fuzzy sets and establishing an inclusion or fuzzy subsethood relationships between them, provides a formal framework to represent clinical knowledge expressing causal relations dealing with imprecision and uncertainty. Based on backward interval-valued binary fuzzy inclusion relations $\overleftarrow{R}_{BSD}$ as interval-valued necessary causal measure and forward interval-valued binary fuzzy inclusion relations as an interval-valued sufficient causal measure, interval-based associative and causal relationships can be established as shown in Figure 1. Domain experts are aware of the complexity of the network of possible relations, but they cannot always deliver exact and consistent values for the strength of some causal relationships, without evaluation a huge account of medical data. They inclined to describe many causal relations by imprecise and vague or interval-based descriptions. For example, some symptoms, which occur obligatory or always with some diseases might not be a confirmation or sufficient for occurring such diseases (e.g., increased serum glucose occurs always with diabetes but does not confirm it, however the causal relationship is strong). This type of causal relationship can be represented by using $\overrightarrow{R}_{FSD}$ and $\overleftarrow{R}_{BSD}$ as a bidirectional interval-valued relation $\overleftrightarrow{R}_{FB}$ as $S_i \xrightleftharpoons[1]{[0.75,1)} D_j$. If there is no information about the strength of a relationship, such relation might be represented as $S_i \xrightleftharpoons[1]{(0,1)} D_j$. On the other hand, the total ignorance about the strength of a relationship might be represented as any value lying within the unit interval, $S_i \xrightleftharpoons[1]{[0,1]} D_j$. In the case of possible occurrence but non-sufficiency, such relation can be represented in less accurate bidirectional interval-valued relation such as $S_i \xrightleftharpoons[(0,1)]{(0,1)} D_j$; e.g., elevated serum amylase sometimes confirm pancreatitis. These relations are still consistent; however, they might need some refinements for the practical use. Furthermore, the binary fuzzy relations specified in CADIAG-IV [3, 13]; e.g., the strength of exclusion and occurrence of with a negated consequent can basically be interpreted as a special case of the $\overrightarrow{R}_{FSD}$ and $\overleftarrow{R}_{BSD}$.

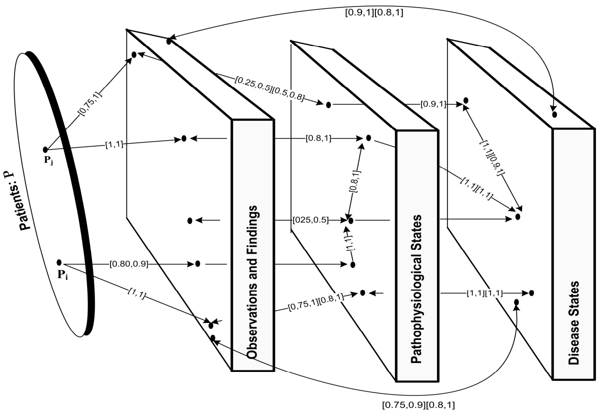

Figure 1. The proposed model.

3.1. Reasoning with Bidirectional Binary Inclusion Relations

Based on the interval-valued binary fuzzy inclusion relations, $\overrightarrow{R}_{FSD}$ and $\overleftarrow{R}_{BSD}$, bidirectional associative and causal relationships, $\overleftrightarrow{R}_{FB}$, can be established. Triangular set of such relations; i.e., locally oriented reasoning, offers a possibility to compute new relations. Before presenting such a computation facility, we will introduce some compulsory definitions describing consistent relationships.

Definition 6: a bidirectional causal relationship $A \xrightleftharpoons[{[\overleftarrow{q}_{min}, \overleftarrow{q}_{max}]}]{[\overrightarrow{q}_{min}, \overrightarrow{q}_{max}]} B$ is dependent if the involved intervals do not violate the inclusion restriction:

$$\frac{\overrightarrow{q}_{min}}{\overleftarrow{q}_{min}}(|A|) = \frac{\overrightarrow{q}_{max}}{\overleftarrow{q}_{max}}(|A|) \quad \text{if } \overrightarrow{q}_{min}, \overleftarrow{q}_{min}, \overrightarrow{q}_{max}, \overleftarrow{q}_{max} \neq 0 \tag{18}$$

$$\text{if } \overrightarrow{q}_{min} = 0 \Rightarrow \overleftarrow{q}_{min} = 0 \tag{19}$$

The first restriction is coming from the following equations:

$\overleftarrow{q}_{max}(|B|) = (|A \cap B|)^{max} = \vec{q}_{max}(|A|)$ for $\overleftarrow{q}_{max} \neq 0$,

$$\overleftarrow{q}_{min}(|B|) = (|A \cap B|)^{min} = \vec{q}_{min}(|A|) \quad (20)$$

and

$$|B| = (\frac{\vec{q}_{min}}{\overleftarrow{q}_{min}})(|A|) = (\frac{\vec{q}_{max}}{\overleftarrow{q}_{max}})(|A|) \quad (21)$$

On the other hand, representing relationship between medical entities arising always with some diseases which might not be a confirmation for occurring such diseases requires a hybrid representation. This type of causal relationship can be represented by using $\vec{R}_{FSD}$ and $\overleftarrow{R}_{BSDp}$ as a bidirectional interval-valued relation such as $S_i \xleftrightarrow[1]{[0.75,1]} D_j$ .To deal with such a type of causal relationship and in the sense of simplification the computation, sometimes it would be desirable to transform such interval valued causal relationships to consistent point valued causal relationships. As a representative value for an interval, the middle of the difference between the upper bound and the lower bound of each interval can be taken i.e., the mean values of a dependent and consistent bidirectional intervals.

Definition 7: a dependent bidirectional causal relationship $A \xleftrightarrow[[\overleftarrow{q}_{min},\overleftarrow{q}_{max}]]{[\vec{q}_{min},\vec{q}_{max}]} B$ can be converted into a consistent point-valued causal relationship in taking the mean values:

$$A \xleftrightarrow[[\overleftarrow{q}_{min},\overleftarrow{q}_{max}]]{[\vec{q}_{min},\vec{q}_{max}]} B \Rightarrow \quad A \xleftrightarrow[\overleftarrow{q}]{\vec{q}} B,$$

where

a. $$\vec{q} = \frac{\vec{q}_{min} + \vec{q}_{max}}{2} \quad (22)$$

b. $$\overleftarrow{q} = \frac{\overleftarrow{q}_{min} + \overleftarrow{q}_{max}}{2} \quad (23)$$

It has to be mentioned, that in the case of not minimal intervals, $\vec{q}$ and $\overleftarrow{q}$ represent approximations for some intervals. Now, based on these properties we can even compute even new reliable values.

Theorem 1: let KB be set of dependent $\vec{R}_{FB_p}$, i.e., forward and backward point-valued binary fuzzy inclusion relations and A, B and C be medical entities then an interval-valued relation can be computed as follows:

$$KB_{point} = \{A \xleftrightarrow[\overleftarrow{y}_1]{\vec{y}_2} B, B \xleftrightarrow[\overleftarrow{z}_1]{\vec{z}_2} C\} \quad A \xrightarrow[[\vec{s}_{min},\vec{s}_{max}]]{} C$$

$$\vec{s}_{min} = \begin{cases} \vec{y}_2 - ((\frac{\vec{y}_2}{\overleftarrow{y}_1})(1-\vec{z}_2)) & \text{if } \vec{z}_2 > 1-\overleftarrow{y}_1, \overleftarrow{y}_1 \neq 0 \\ 0 & \text{otherwise} \end{cases} \quad (24)$$

$$\vec{s}_{max} = \begin{cases} Min(\vec{y}_2, (\frac{\vec{z}_2\vec{y}_2}{\overleftarrow{y}_1})) + \\ Min((1-\vec{y}_2), ((\frac{\vec{z}_2\vec{y}_2}{\overleftarrow{z}_1\overleftarrow{y}_1})(1-\overleftarrow{z}_1))) \\ \quad \text{if } \overleftarrow{z}_1, \overleftarrow{y}_1 \neq 0 \\ 0 \quad \text{if } \overleftarrow{z}_1 = 1, \overleftarrow{y} = \vec{y}_2 = 0 \\ 1-\vec{y}_2 \quad \text{Otherwise} \end{cases} \quad (25)$$

Based on definition 1 (scalar cardinality) , the additivity property, definition 2, and dependency property def 6, the grades of the involved compound bidirectional inclusion relationships in such KB can be rewritten in terms of equalities and inequalities. The inclusion degrees between A and C can be computed comparatively to the cardinality of $|A|$. The minimal and maximal cardinalities of $|A \cap C|$ are consistent to $(|A \cap B|)^{min}, (|A \cap B|)^{max}, (|B \cap C|)^{min}, (|B \cap C|)^{max}$, and to $|B|,|C|$, which can be expressed in terms of relative cardinality of $|A|$. The computed values are locally consistent to the KB; i.e., the intervals do not contain inconsistent values. For example, the following holds in the minimal case of $\vec{s}$:

$$\vec{s}_{min} = \frac{(|A \cap B \cap C|)^{min}}{|A|} > 0 \Leftrightarrow$$

$$(|B \cap C|)^{min} > |B| - (|A \cap B|)^{min} \Leftrightarrow \vec{r}_{min} > 1 - \overleftarrow{q}_{min}, \quad (26)$$

otherwise $\vec{s}_{min}$ =0 , and so forth.

Examples:

- Traditional implication relation:

  $$\{S_1 \xleftrightarrow[1]{1} D_1 \xleftrightarrow[1]{1} D_2\} \overset{Theorem\ 1}{\Rightarrow} S_1 \xrightarrow[1]{} D_2$$

  This example, represents the classical case; i.e., the two valued or the material implication relation as special case of this model. The fuzzy set $S_1 \subset D_1$ and $D_1 \subset S_1$ and therefore $S_1 \subset D_2$, i.e., all patients having the symptom S_1 must have D_2.
- Derivation of new relations:

  $$\{S_1 \xleftrightarrow[1]{0.75} D_3 \xleftrightarrow[0.75]{[0.5,1]} D_4\} \overset{Def.\ 6,7}{\Rightarrow}$$

  $$\{S_1 \xleftrightarrow[1]{0.75} D_3 \xleftrightarrow[0.75]{[0.75]} D_4\} \overset{Theorem\ 1}{\Rightarrow}$$

  $$S_1 \xrightarrow[[0.56,0.75]]{} D_4$$

  Based on utilizing the defuzzification and theorem 1, we can even derive new relations.
- Compound binary fuzzy relation and the defuzzification property:

  $$\{S_1 \xleftrightarrow[[0.5,1]]{[0.5,0.75]} D_1 \xleftrightarrow[[0.5,1]]{[0.75,1]} D_5\} \overset{Def.\ 6,7}{\Rightarrow}$$

  $$\{S_1 \xleftrightarrow[[0.75]]{[0.62]} D_1 \xleftrightarrow[[0.75]]{[0.87]} D_5\} \overset{Theorem\ 1}{\Rightarrow}$$

  $$S_1 \xrightarrow[[0.52,0.87]]{} D_5$$

Based on the above mentioned theorem the obtained binary interval based fuzzy relations correspond to an approximation of an interval valued computation for a knowledge base.

4. Discussion

Knowledge acquisition is considered to be crucial in constructing medical knowledge based systems. Extracting precise values for representing uncertainty and or imprecision could be unavailable or hard to get. Furthermore, verifying knowledge bases for logical consistency is essential in ensuring quality of a medical system. This work attempts therefore to handle these crucial aspects in presenting a wide-ranging model for medical knowledge representation relying on consistent interval propagation expressing the uncertainty about causal and associational relationships at different levels of the clinical knowledge.

The focus of attention of this work is based on utilizing the fuzzy inclusion measure as departure point to express causal and associational relations between medical entities in the form of interval-valued or pointvalued necessary and sufficient causal measures. In this context, some important medical measures, such as necessary and sufficiency measures have been formalized in the light of reinterpretation of the fuzzy inclusion relationship. Furthermore, a computational model has been presented within the concept of fuzzy inclusion measure between medical entities.

The results of this model are promising, in the sense, that the computed intervals are consistent, and can be refined. On the other hand, considering negative relationships between medical entities such as interval valued necessary and sufficiency measures between medical entities, e.g., degree($\alpha \subset \neg\beta$) and the negative occurrence as the degree($\neg\beta \subset \alpha$), might be desirable in the reasoning process. These aspects could be regarded as special cases of this model but they need further analysis.

As mentioned earlier, there is a corresponding similarity between this model and the knowledge acquisition component of the CADIAG-Systems project [3] that can partly be regarded as a special case of this model. Even more, the presented concept of forward and backward inclusion relation measure has a semantic connection to the statistical terms used in project (detection and correction of non word in Arabic project) such as Arabic Root PredictiVe (RPV) and Pattern Predictive Value (PPV). Details are found in [9].

Finally, for future work, an integration of an interval based compositional rule of inference in the sense of utilizing the compound interval-based causal measure can be regarded as an enhancement towards a reasoning method based on consistent interval propagation

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