A General Characterization of Representing and Determining Fuzzy Spatial Relations

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Abstract: A considerable amount of fuzzy spatial data emerged in various applications leads to investigation of fuzzy spatial data and their fuzzy relations. Because of complex requirements, it is challenging to propose a general fuzzy spatial relationship representation and a general algorithm for determining all fuzzy spatial relations. This paper, presents a general characterization of representing fuzzy spatial relations assuming that fuzzy spatial regions are all fuzzy. On the basis of it, correspondences between fuzzy spatial relations and spatial relations are investigated. Finally, a general algorithm for determining all fuzzy spatial relations is proposed.

Keywords: Fuzzy spatial data, fuzzy point, fuzzy line, fuzzy region, fuzzy spatial relations.

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1. Introduction

Spatial relations play a fundamental role in various application areas, ranging from Geographic Information System (GIS) systems [10] to image understanding [6]. It can be divided into topological (e.g., “overlap”, “meet”, etc.), directional (e.g., “north of”, “south of”, etc.), and metric (e.g., “3km away from”, etc.) relations [2]. Due to topological relations having great significances to spatial reasoning and spatial analysis, it has received increasing attentions in a quantitative way: Both the Region Connection Calculus (RCC) [8] and the 9-intersection model [3] provide a formal characterization of qualitative spatial relations.

However, spatial data is usually fuzzy in the real world applications since their values are subjective in real applications [1, 7]. Thus, the problems that emerge are how fuzzy spatial data should be modeled to determine their fuzzy topological relations [13]. To fill this gap, various definitions of spatial relations between fuzzy spatial data extend either the RCC or the 9-intersection model by considering a FR as being composed of two components [9]: One consists of the points that are definitely in FR and one consists of the points that are definitely not in the fuzzy region.

A straightforward Fuzzification of the definitions in RCC-8 relations is proposed in [5]. Esterline et al. [5] presented a fuzzy version of crisp spatial logic developed by Randell et al. [8] that takes the single relation connected-with as primitive. Unfortunately, many properties of the original RCC-8 relations are lost. Moreover, it is unclear how to apply definitions proposed in [5] to calculate the values of the fuzzy spatial relations between two given fuzzy regions. In order to solve these problems, Schockaert et al. [12] extend the RCC by providing generalized definitions of the spatial relations as fuzzy relations, which allows expressing the degree to which a particular spatial relation between two regions holds.

Concerning on fuzzification of 9-intersection model, several researches extend 9-intersection approach based on the interior, boundary and exterior of the simple fuzzy spatial regions [14, 15, 16]. Tang et al. [16] studied definitions in fuzzy boundary and their relations and then extend the 9-intersection approach to the 3*3 intersection matrix. Furthermore, 4*4 intersection matrices are formalized based on different topological parts of two fuzzy sets in [14]. In [15] a framework for dealing with fuzzy spatial objects was theoretically proposed, which was also compatible with non-fuzzy spatial object.

However, to our best knowledge, there are less reports on correspondences between fuzzy spatial relations and spatial relations more specifically from mathematical point of view although fuzzy spatial relations have been formalized in the fields of both RCC [5, 12] and 9-intersection model [14, 15, 16] and less reports on general algorithm for determining all fuzzy relations (there are totally 23 fuzzy relations and it will be presented in the later section in this paper) although specific methods of that [12, 16] has been proposed. In this paper, we propose a general characterization of representing and determining fuzzy spatial relations from mathematical point of view assuming that fuzzy spatial regions are all fuzzy. We firstly present the basics of representation of fuzzy spatial topological relations from three aspects: Fuzzy Point (FP), Fuzzy Line (FL) and Fuzzy Region (FR) and then give definitions of fuzzy relations. On this basis, correspondences between fuzzy spatial topological relations and spatial topological relations...
are investigated. Finally, a general algorithm for determining fuzzy relations is proposed.

The rest of the paper is organized as follows: Section 2 presents basics of representation of fuzzy spatial data and gives definitions of fuzzy spatial topological relations. Section 3 investigates correspondences between fuzzy spatial relations and spatial relations. Section 4 proposes a general algorithm for determining fuzzy relations and Section 5 concludes the paper.

2. Representation of Fuzzy Spatial Topological Relations

In this section, we present the basics of representation of fuzzy spatial topological relations from three aspects, which are FP, FL and fuzzy region. Fuzzy relations, which will be mentioned below, indicate fuzzy spatial topological relations for simplicity.

2.1. Fuzzy Point

A FP is a point whose exact position is not determined but possible positions are known within a certain area. In that case, a FP can be viewed as a point in two-dimensional Euclidean space using a membership degree, which returns the value of its membership function, to indicate possible positions of FP.

- **Definition 1. (FP):** For a FP (denoted as FP), we have $FP=(x, y, \delta)$, including:
  - $x$ is the projection value of the position to $x$-axis.
  - $y$ is the projection value of the position to $y$-axis.
  - $\delta$ is the membership degree of the point being the position $(x, y)$ (denoted as $\delta_{xy}$), where $0 \leq \delta \leq 1$.

A FP $(x_0, y_0, \delta_0)$ indicates that the possibility of the point locating at $(x_0, y_0)$ is $\delta_0$. For example, a FP $(2, 5, 0.8)$ indicates the possibility of the point locating at $(2, 5)$ is 0.8. It is noted that if $\delta_{xy}=1$ and 0 otherwise, the point $(x, y, \delta)$ is a crisp one.

The fuzzy relations of two FPs contain two cases, which are fuzzy equal (denoted as FP\text{equal}) and fuzzy disjoint (denoted as FP\text{disjoint}). In the following definition, we will define each of them. Here, we introduce a mathematical symbol $supp$, where $supp A=\{u | u \in U, A(u) > 0\}$.

- **Definition 2. Fuzzy Relations of FPs:** For two FPs $FP_1=(x_1, y_1, \delta_1)$ and $FP_2=(x_2, y_2, \delta_2)$, we have:
  - $FP$\text{equal} ($FP_1, FP_2$): $\exists supp (x_1, \delta_1)=supp(x_2, \delta_2) \cdot supp(y_1, \delta_1)=supp(y_2, \delta_2)$.
  - $FP$\text{disjoint} ($FP_1, FP_2$): $\forall (\neg (supp(x_1, \delta_1)=supp(x_2, \delta_1) \cdot supp(y_1, \delta_1)=supp(y_2, \delta_2))$.

2.2. Fuzzy Line

A FL is a line whose exact position or length is unknown but the area the line ranges is known. The semantic of a line is a point set between two ending points. Accordingly, a FL can be viewed as a line in two-dimensional Euclidean space using two membership degrees, which return values of two ending points’ membership functions, to indicate possible positions of the fuzzy line.

- **Definition 3. Fuzzy Line:** For a FL (denoted as FL), we have $FL=(x_0, y_0, \delta_0, x_1, y_1, \delta_1)$, including:
  - $x_0$ and $y_0$ are the minimum projection values of the FL to $x$-axis and $y$-axis (left ending point).
  - $x_1$ and $y_1$ are the maximum projection values of the FL to $x$-axis and $y$-axis (right ending point).
  - $\delta$ and $\delta'$ are the membership degrees of the two ending points, where $0 \leq \delta \leq 1$ and $0 \leq \delta' \leq 1$.

Since, FL is determined by two fuzzy ending points, the membership degree of FL is actually determined by the membership degrees of those two ending points. The fuzzy relations of a FP and a FL contain three cases, which are fuzzy meet (denoted as FL\text{meet}), fuzzy contain (denoted as FL\text{contain}), and fuzzy disjoint (denoted as FL\text{disjoint}). Definitions of them are given in the following.

- **Definition 4. Fuzzy Relations of FP and Fuzzy Line:** For a FP, $FP=(x_0, y_0, \delta_0)$ and a FL, $FL=(x_0, y_0, \delta_0, x_1, y_1, \delta_1)$, we have:
  - FL\text{meet} ($FL, FP$): $\neg supp (x_0, y_0, \delta_0) \cap supp (x_0, y_0, \delta) = \emptyset$ \land $\neg supp (x_0, y_0, \delta_0) \cap supp (x_1, y_1, \delta_1) = \emptyset$.
  - FL\text{contain} ($FL, FP$): $\exists (\neg supp (x_0, y_0, \delta_0) \cap supp (x_1, y_1, \delta_1) = \emptyset) \land (\neg supp (x_0, y_0, \delta_0) \cap supp (x_1, y_1, \delta_1) = \emptyset) \land \neg supp (x_0, y_0, \delta_0) \cap supp (x_1, y_1, \delta_1) = \emptyset$.
  - FL\text{disjoint} ($FL, FP$): $\neg (FL\text{meet} (FL, FP) \lor FL\text{contain}(FL, FP)) \Rightarrow FP$ stays in the left of $FL\Leftrightarrow \exists y_0, y_1 \mid y_0 > y_1 \Rightarrow$ denoted as FL\text{left} ($FP, \delta_0, FL, \delta, \delta'$) \lor FP stays in the right of $FL\Leftrightarrow \exists y_0, y_1 \mid y_0 < y_1 \Rightarrow$ denoted as FL\text{right} ($FP, \delta_0, FL, \delta, \delta'$).

The fuzzy relations of two fuzzy lines contain six cases, which are fuzzy intersect (denoted as FL\text{intersect}), fuzzy equal (denoted as FL\text{equal}), fuzzy contain (denoted as FL\text{contain}), fuzzy overlap (denoted as FL\text{overlap}), fuzzy meet (denoted as FL\text{meet}), and fuzzy disjoint (denoted as FL\text{disjoint}).

- **Definition 5. Fuzzy Relations of Fuzzy Lines:** For two fuzzy lines $FL_1=(x_1, y_1, \delta_1, x_1, y_1, \delta_1)$ and $FL_2=(x_2, y_2, \delta_2, x_2, y_2, \delta_2)$, we have:
  - FL\text{intersect} ($FL_1, FL_2$): $(FL\text{left} (x_1, y_1, \delta_1, FL_2) \land FL\text{right} (x_2, y_2, \delta_2, FL_1)) \lor FL\text{left} (x_2, y_2, \delta_2, FL_1) \land FL\text{right} (x_1, y_1, \delta_1, FL_2)) \lor (FL\text{left} (x_2, y_2, \delta_2, FL_1) \land FL\text{right} (x_1, y_1, \delta_1, FL_2) \land FL\text{left} (x_1, y_1, \delta_1, FL_2) \land FL\text{right} (x_2, y_2, \delta_2, FL_1)) \lor FL\text{right} (x_1, y_1, \delta_1, FL_2) \land FL\text{right} (x_2, y_2, \delta_2, FL_1)$.
2.3. Fuzzy Region

A general definition describes a crisp region as a set of disjoint, connected components, called faces, possibly with disjoint holes [4, 11] in the Euclidean space IR^2. By analogy with the generalization of crisp regions to fuzzy regions, we strive for fuzzy regions on the basis of the point set paradigm and fuzzy concepts. For simplicity, we only talk about two-dimensional regions without holes.

A FR is a region with indeterminate boundaries. Fuzzy regions can be represented by MBR (minimum bounding rectangle) so that we can use two FPs to represent fuzzy regions.

**Definition 6. Fuzzy Region:** For a FR (denoted as FR), we have $FR = (x_{min}, y_{min}, \delta, x_{max}, y_{max}, \delta')$, including:

- $x_{min}$ and $y_{min}$ are the minimum projection values of the FR to x-axis and y-axis (lower left ending point).
- $x_{max}$ and $y_{max}$ are the maximum projection values of the FR to x-axis and y-axis (upper right ending point).
- $\delta$ and $\delta'$ are the membership degrees of the above two representing points, where $0<\delta \leq 1$ and $0<\delta' \leq 1$.

Similar as the fuzzy line, the membership degree of FR is determined by the membership degrees of the two representing ending points.

The fuzzy relations of a FP and a FR contain three cases, which are fuzzy disjoint (denoted as FRPdisjoint), fuzzy meet (denoted as FLPmeet) and fuzzy contain (denoted as FRPcontain). Definitions of them are given in the following.

**Definition 7. Fuzzy Relations of FP and Fuzzy Region:** For a FP, $FP = (x, y, \delta)$ and a FR, $FR = (x_{min}, y_{min}, \delta, x_{max}, y_{max}, \delta')$, we denote four FL of the fuzzy region: $FL_1 = (x_{min}, y_{max}, \delta_1, x_{max}, y_{max}, \delta_2)$, $FL_2 = (x_{min}, y_{max}, \delta_1, x_{min}, y_{min}, \delta_2)$, $FL_3 = (x_{max}, y_{min}, \delta_3, x_{max}, y_{max}, \delta_2, \delta_3)$, $FL_4 = (x_{max}, y_{min}, \delta_3, x_{min}, y_{min}, \delta_2)$. Then, we have:

- FLPdisjoint (FP, FR): $\neg(FLPleft(FP, FR) \land FLPright(FP, FR))$.
- FLPmeet (FP, FR): $FLPmeet(FP, FR) \lor (FLPleft(FP, FR) \land FLPright(FP, FR))$.
- FLPcontain (FP, FR): $FLPcontain(FP, FR) \lor (FLPleft(FP, FR) \land FLPright(FP, FR))$.

The fuzzy relations of a FL and a FR contain four cases, which are fuzzy contain (denoted as FRPcontain), fuzzy intersect (denoted as FLPmeet), fuzzy meet (denoted as FLPmeet) and fuzzy disjoint (denoted as FLPdisjoint). Definitions of them are given in the following.

**Definition 8. Fuzzy Relations of FL and Fuzzy Region:** For a FL, $FL = (x_1, y_1, \delta_1, x_2, y_2, \delta_2)$ and a FR, $FR = (x_{min}, y_{min}, \delta_1, x_{max}, y_{max}, \delta_2)$, we denote four FL of the fuzzy region: $FL_1 = (x_{min}, y_{max}, \delta_1, x_{max}, y_{max}, \delta_2)$, $FL_2 = (x_{min}, y_{max}, \delta_1, x_{min}, y_{min}, \delta_2)$, $FL_3 = (x_{max}, y_{min}, \delta_3, x_{max}, y_{max}, \delta_3, \delta_2)$, $FL_4 = (x_{max}, y_{min}, \delta_3, x_{min}, y_{min}, \delta_2)$. Then, we have:

- FLPcontain (FR, FL): $(FLPcontain(FR, FL) \land FLPcontain(FR, FR) \land FLPmeet(FR, FR))$.
- FLPmeet (FR, FL): $FLPmeet(FR, FL) \lor (FLPleft(FR, FL) \land FLPright(FR, FL))$.
- FLPcontain (FR, FL): $FLPcontain(FR, FL) \lor (FLPleft(FR, FR) \land FLPright(FR, FR))$.

The fuzzy relations of two fuzzy regions contain five cases, which are fuzzy equal (denoted as FLPequal), fuzzy contain (denoted as FRPcontain), fuzzy overlap (denoted as FLPoverlap), fuzzy meet (denoted as FLPmeet) and fuzzy disjoint (denoted as FRPdisjoint). Definitions of them are given in the following:

**Definition 9. Fuzzy Relations of Fuzzy Regions:** For two fuzzy regions $FR_1 = (x_{min}, y_{min}, \delta_1, x_{max}, y_{max}, \delta_2)$ and $FR_2 = (x_{min}, y_{min}, \delta_1, x_{max}, y_{max}, \delta_2)$, we denote four FL of the fuzzy region: $FL_1 = (x_{min}, y_{min}, \delta_1, x_{max}, y_{max}, \delta_2)$, $FL_2 = (x_{min}, y_{min}, \delta_1, x_{min}, y_{min}, \delta_1)$, $FL_3 = (x_{max}, y_{min}, \delta_3, x_{max}, y_{max}, \delta_3)$, $FL_4 = (x_{max}, y_{min}, \delta_3, x_{min}, y_{min}, \delta_3)$, $FL_5 = (x_{max}, y_{min}, \delta_3, x_{max}, y_{max}, \delta_3, \delta_2)$, $FL_6 = (x_{max}, y_{min}, \delta_3, x_{min}, y_{min}, \delta_2, \delta_3)$. Then, we have:

- FLPequal (FR_1, FR_2): $FLPequal(FR_1, FR_2) \land (FLPleft(FR_1, FR_2) \land FLPright(FR_1, FR_2))$. 
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3. Correspondences between Fuzzy Spatial Topological Relations and Spatial Topological Relations

In this section, we present correspondences between fuzzy spatial topological relations and spatial topological relations on the basis of studies in the above section. The correspondences come from six cases: Point and point, line and point, line and line, region and point, region and line, region and region.

For fuzzy relations of FP, the fuzzy relation is if there are possible equal points and FPDisjoint is the fuzzy relation if there are no possible equal points.

For fuzzy relations of a FP and a fuzzy line, FPMeet is the fuzzy relation if there is a possible point meeting a possible line; FPContain is the relation if there is a possible line containing possible points and there is no possible point meeting a possible line; FPLDisjoint is the relation if all possible points and all possible lines are disjoint.

For fuzzy relations of two fuzzy lines, FLIntersect is the fuzzy relation if there is a possible line of one FL intersecting a possible line of the other fuzzy line; FLLEqual is the fuzzy relation if there are two possible equal lines of two fuzzy lines; FLLContain is the fuzzy relation if the minimum and maximum ending points of a possible line of one FL are contained by a possible line of the other fuzzy line; FLLOverlap is the fuzzy relation if the maximum ending point of a possible line of one FL is contained by a possible line of the other FL and the minimum ending point of a possible line of the other FL is contained by a possible line of this fuzzy line; FLLMeet is the fuzzy relation if there is a ending point of a possible line of one FL meeting a ending point of a possible line of the other fuzzy line; FLLDisjoint is the fuzzy relation if all the two possible lines of two fuzzy lines are disjoint.

For fuzzy relations of a FP and a fuzzy region, it is FPRLDisjoint if all possible points of the FP and all possible regions of FR are disjoint; it is FRPMeet if there is a possible point of the FP meeting a possible region of FR and not all possible points of FP staying outside or inside all possible regions of the fuzzy region; it is FRPContain if all possible points of FP are contained by all possible regions of the fuzzy region.

For fuzzy relations of a FL and a fuzzy region, it is FRContain if all possible lines of FL are contained by all possible regions of the fuzzy region; it is FLRLDisjoint if all possible lines of FL and all possible regions of FR are disjoint; it is FLRLMeet if there is a possible line of FL meeting a possible region of FR and their fuzzy relation is not FRLContain or FLRLDisjoint; it is FLRLIntersect if there is a possible line of FL intersecting a possible region of FR and their fuzzy relation is not FRLcontain or FRLDisjoint or FRLMeet.

For fuzzy relations of two fuzzy regions, it is FRRLDisjoint if all possible regions of one FR and all possible regions of the other FR are disjoint; it is FRRLContain if all possible regions of one FR are contained by all possible regions of the other fuzzy region; it is FRRLMeet if there is a possible region of one FR overlaps a possible region of the other FR and their fuzzy relation is not FRRLDisjoint, FRRLcontain or FRRLoveralp; it is FRRLEqual if a possible region of one FR and a possible region of the other FR are equal and their fuzzy relation is not FRRLdisjoint, FRRLcontain, FRRLoveralp or FRRLmeet.

In order to present correspondences between fuzzy spatial topological relations and spatial topological relations, Figure 1 shows their correspondences. In Figure 1, it is denoted as cross if there are no correspondences; it is denoted as tick if there are correspondences and the fuzzy relation has the corresponding crisp relation; it is blank if there are correspondences but the fuzzy relation has no corresponding crisp relation.

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Figure 1. Correspondences between fuzzy and crisp spatial relations.
4. Determination of Fuzzy Spatial Topological Relations

In this section, we present how to determine fuzzy spatial topological relations. We firstly propose a general algorithm for determining fuzzy relations, and then an example is given to explain it.

There are 23 fuzzy relations. Each of them needs an algorithm to determine fuzzy relations. Since, there are some common points in each of them, we propose a general algorithm. Then, if a certain fuzzy relation is required, the general algorithm can be extended.

The Algorithm 1 is a general algorithm for determining fuzzy relations. It contains two loops in order to compare all possible fuzzy relations. The possibility of the relation employs cumulative way to compute. The finally returned value is divided by all possibility of the relation employs cumulative way to compute. The finally returned value is divided by all possible fuzzy relations. The general algorithm can be extended.

Algorithm 1: Frelation Y, Z.

1. for \( k = 1 \); \( k <= X; k++ \)
2. let \( \delta_k = 0 \)
3. end for
4. for \( m = 1; m <= i; i++ \)
5. for \( n = 1; n <= m; n++ \)
6. for \( r = 1; r <= X; r++ \)
7. if Frelation \( Y, Z \)
8. \( \delta_i = \delta_i + \delta_r \)
9. end for
10. end for
11. end for
12. if true (Frelation)
13. return \( \delta_T / \sum_{i=1}^{X} \delta_i \)

For the algorithm, some tips need to be explained: \( Y \) and \( Z \) indicate two fuzzy spatial objects (it can be further explained by their representing points); \( X \) indicates number of fuzzy relations between \( Y \) and \( Z \). \( F;relation \( Y, Z \) \) compares fuzzy relations \( X \) times according to definition 2, 4, 5, 7, 8, or 9; \( T \) indicates the number satisfying fuzzy relation ranging from 1 to \( X \), and the returned value is divided by all the possible membership degrees.

In succession, we give an example to describe process of determining fuzzy relations using the proposed algorithm. For convenience, we use the example of two fuzzy regions.

Consider two fuzzy regions \( FR_1 = \{(1, 2, 0.6, 6, 7, 0.7), (2, 4, 0.4, 7, 8, 0.3)\} \) and \( FR_2 = \{(6, 3, 0.8, 9, 10, 0.6)\} \). According to line 10 in algorithm FRRrelation, we can get the fuzzy relation of \( FR_1 \) and \( FR_2 \) is FRRoverlap. The meet pair are \( \{(1, 2, 0.6, 6, 7, 0.7), (6, 3, 0.8, 9, 10, 0.6)\} \), \( \{(1, 2, 0.6, 6, 7, 0.7), (6, 1, 0.2, 9, 3, 0.4)\} \); the overlap pair is \( \{(2, 4, 0.4, 7, 8, 0.3), (6, 3, 0.8, 9, 10, 0.6)\} \); the disjoint pair is \( \{(2, 4, 0.4, 7, 8, 0.3), (6, 1, 0.2, 9, 3, 0.4)\} \). Possibility of each relation is: \( \delta_T = 0.7 \times 0.2 \times 0.4 = 0.2352 \); \( \delta_1 = 0.4 \times 0.3 \times 0.8 \times 0.6 = 0.0576 \); \( \delta = 0 + 0.4 \times 0.3 \times 0.2 \times 0.4 = 0.0096 \). Finally, we get the possibility of each relation: \( \delta_1 \left( \delta_1 + \delta_2 + \delta_3 + \delta_4 + \delta_5 \right) \approx 0.778 \); \( \delta_2 \left( \delta_1 + \delta_2 + \delta_3 + \delta_4 + \delta_5 \right) \approx 0.190 \); \( \delta_1 \left( \delta_1 + \delta_2 + \delta_3 + \delta_4 + \delta_5 \right) \approx 0.032 \). Consequently, the possibility that the relation of the two fuzzy regions is meet approximately amounts to 0.778; is overlap approximately amounts to 0.190; is disjoint approximately amounts to 0.032.

5. Conclusions

In order to present a general characterization of representing and determining fuzzy spatial relations, definitions of fuzzy spatial objects and their relations are given. Then, correspondences between fuzzy and crisp spatial relations are investigated. Finally, a general algorithm for determining all fuzzy relations is proposed and a followed example explains it. Compared with other methods, our approaches focus on correspondences between fuzzy spatial relations and spatial relations more specifically from mathematical point of view, which has less been studied. What’s more, a general formal algorithm for determining fuzzy relations is proposed, while majority of others are methods for specific domains. Consequently, our approaches can be applied to more applications than others.

In the future, we intend to apply the proposed approaches to spatiotemporal applications. A possible solution is to integrate our approaches with MBR strategies. Another future research topic is extending two-dimensional spatial data to three-dimensional one, and discussing their continuous cases.

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