Distinguishing Attack on Common Scrambling Algorithm

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Abstract: Common Scrambling Algorithm (CSA) has been used to encrypt european cable digital television signals since 1994. Although, the key size of CSA is small, up to now, there haven’t any effective crypto results which can break the algorithm totally. Based on the idea of slide resynchronization attack, a distinguishing attack which can distinguish the keystream of the Stream Cipher (SC) from a purely random sequence with computational complexity of $O(2^{15})$ is proposed. According to the distinguishing attack, the 64 bit initial key can be recovered with computational complexity of $O(2^{55})$.

Keywords: Digital video broadcasting common scrambling algorithm, distinguishing attack, slide resynchronization attack, hybrid cipher.

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1. Introduction

The Digital Video Broadcasting Common Scrambling Algorithm (DVB-CSA) has been used to encrypt european cable digital television signals since 1994. As part of the MPEG-TS standard, it is virtually used for all conditional access systems in digital television. The algorithm has never been officially revealed and was only available under a non-disclosure agreement from an ETSI custodian before 2002. The agreement didn’t permit the algorithm to be implemented in software for “security reasons”. In 2002, a software program called FreeDec appeared on the internet, which implemented CSA in software, then the algorithm was reverse engineered quickly by hackers and the details of the CSA was accessed to the public.

Weinmann and Wirt [10] proposed a guess and determine attack [8] with complexity less than $2^{45}$. Wirt [11] presented a fault attack on the block cipher which can be launched without regarding the Stream Cipher part (SC). Although, the complexity of this attack is within seconds, the condition of the attack model is too strong to be compared. In [8] Simpson figured out some mistakes in the description of the SC part of CSA and presented time-memory tradeoff attacks on the stream cipher, the results are as follows [10]:

<table>
<thead>
<tr>
<th></th>
<th>Data</th>
<th>Memory</th>
<th>Time</th>
</tr>
</thead>
<tbody>
<tr>
<td>State Recovery</td>
<td>$2^{20}$</td>
<td>$2^{15}$</td>
<td>$2^{20}$</td>
</tr>
<tr>
<td>Key Recovery</td>
<td>$2^{41.5}$</td>
<td>$2^{25}$</td>
<td>$2^{31}$</td>
</tr>
</tbody>
</table>

Twes et al. [9] presented a time memory trade off attack against 48 bit key CSA, the result showed that CSA can be broken in real time if the precomputed tables are available and calculating these precomputations need over years and the author claimed that “when the algorithm using 64 instead of 48 independent bits for a key would render time tradeoffs inefficient”, this is another tradeoff method for CSA cryptanalysis.

Distinguishing attack was firstly proposed in [1]. In a distinguishing attack, the adversary tries to judge whether a given sequence stems from a known cipher or a purely random sequence. A distinguishing attack may not be as strong as a key-recovery attack, but it can provide some undesired information leakage to the adversary. Related key cryptanalysis was first introduced by Biham [3], it is a type of chosen key attacks, in which the relationship between the keys used is known. The slide resynchronization attack was firstly proposed on the cryptanalysis of Grain [6] which is a related key attack using the slide property of the key. This cryptanalytic tool shows great potential in cryptanalysis for some stream ciphers [2, 4, 5, 7, 12] whose initialization phase and key stream generating phase are similar.

In this paper, we propose a distinguishing attack at the basis of “slide resynchronization attack” which can distinguish the key stream of the SC part from a purely random sequence with computational complexity of $O(2^{15})$. According to the distinguishing attack, we can recover the full 64 bit key with computational complexity of $O(2^{55})$.

2. Description of CSA

CSA is a typical hybrid cipher which can be seen as a cascade of a block cipher and a SC. Both ciphers use the same 64 bit key K, which is called the common key. The following description of CSA refers the descriptions in reference [8, 10]. For scrambling, the plaintext is divided into blocks of 64 bit length (8 byte) and the sequence of blocks is encrypted with a custom block cipher in CBC mode using reverse
order, then the ciphertext of the block cipher are encrypted with the key stream generated by the stream part of the cipher, the residue is directly XORed to the key stream of the SC and the last output of the block cipher is used as IV for the SC. Figure 1 depicts in the scrambling process how the block and the SC are combined.

\[ A'_0 = \begin{cases} \ A_0 \oplus X \oplus D \oplus I^A & \text{Initialization} \\ \ A_0 \oplus X & \text{Keystream generating} \end{cases} \] (2)

The new state of register \( A \) is \( A' = (A'_0, A_9, \ldots, A_8) \).

2. The register \( B \) is updated as follows:

\[ B'_0 = \begin{cases} \ B_0 \oplus B_9 \oplus Y \oplus I^B & \text{Initialization} \\ B_0 \oplus B_9 \oplus Y & \text{Keystream generating} \end{cases} \] (3)

The new state of register \( B \) is as follows:

\[ B' = \begin{cases} (B'_0, B_0, \ldots, B_8) & p = 0 \\ (\text{rot}(B'_0), B_0, \ldots, B_8) & \text{else} \end{cases} \] (4)

3. The registers \( X, Y, Z, p \) and \( q \) are updated through Boolean functions of \( F_i \), \( 0 \leq i \leq 13 \), which can be seen as S-Boxes, the input variables and corresponding \( F_i \) are shown in Table 2 below:

Table 2. Corresponding input and output variables of S-Boxes.

<table>
<thead>
<tr>
<th>S-Box</th>
<th>Input Variables</th>
<th>Output</th>
</tr>
</thead>
<tbody>
<tr>
<td>( S_0 )</td>
<td>( s_6, s_5, s_4, s_3 )</td>
<td>( F_1(s_6) )</td>
</tr>
<tr>
<td>( S_1 )</td>
<td>( s_6, s_5, s_4, s_2 )</td>
<td>( F_2(s_4) )</td>
</tr>
<tr>
<td>( S_2 )</td>
<td>( s_6, s_5, s_4, s_3 )</td>
<td>( F_3(s_6) )</td>
</tr>
<tr>
<td>( S_3 )</td>
<td>( s_6 )</td>
<td>( F_4(s_6) )</td>
</tr>
</tbody>
</table>

The input variables of the S-Boxes are corresponding states of register \( A \) as follows:

Table 3. Relationship between the internal states and the input of Boolean functions.

<table>
<thead>
<tr>
<th>Input Variable</th>
<th>( X_0 )</th>
<th>( X_1 )</th>
<th>( X_2 )</th>
<th>( X_3 )</th>
<th>( X_4 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( s_0 )</td>
<td>( A_{4,0} )</td>
<td>( A_{4,0} )</td>
<td>( A_{4,0} )</td>
<td>( A_{4,0} )</td>
<td>( A_{4,0} )</td>
</tr>
<tr>
<td>( s_1 )</td>
<td>( A_{4,1} )</td>
<td>( A_{4,1} )</td>
<td>( A_{4,1} )</td>
<td>( A_{4,1} )</td>
<td>( A_{4,1} )</td>
</tr>
<tr>
<td>( s_2 )</td>
<td>( A_{4,3} )</td>
<td>( A_{4,0} )</td>
<td>( A_{4,0} )</td>
<td>( A_{4,0} )</td>
<td>( A_{4,0} )</td>
</tr>
<tr>
<td>( s_3 )</td>
<td>( A_{4,3} )</td>
<td>( A_{4,1} )</td>
<td>( A_{4,1} )</td>
<td>( A_{4,2} )</td>
<td>( A_{4,2} )</td>
</tr>
<tr>
<td>( s_4 )</td>
<td>( A_{4,2} )</td>
<td>( A_{4,3} )</td>
<td>( A_{4,0} )</td>
<td>( A_{4,1} )</td>
<td>( A_{4,2} )</td>
</tr>
<tr>
<td>( s_5 )</td>
<td>( A_{4,3} )</td>
<td>( A_{4,2} )</td>
<td>( A_{4,0} )</td>
<td>( A_{4,2} )</td>
<td>( A_{4,3} )</td>
</tr>
<tr>
<td>( s_6 )</td>
<td>( A_{4,1} )</td>
<td>( A_{4,0} )</td>
<td>( A_{4,0} )</td>
<td>( A_{4,0} )</td>
<td>( A_{4,3} )</td>
</tr>
</tbody>
</table>

The algebraic normal forms 5-input boolean functions of \( F_i \), \( 0 \leq i \leq 13 \) are presented in the appendix of reference [8].

4. The update of FSM: The FSM include registers \( E, F \) and carry bit Register \( c \), they are updated as follows:

\[ (E, F, c) = \begin{cases} \ (q \neq 0) & \ (F, c) \\ \ (F, (E + Z \oplus c) \mod 2^4) & \text{else} \end{cases} \] (5)
If \( q = 0 \), \( c \) is unchanged. Otherwise, it is 1 if \( E + F + c \geq 2^4 \)
and 0 else.

5. The output: The SC generates 2 bit-key stream each
clock cycle, i.e., \( D_2 \oplus D_3 \oplus D_0 \oplus D_1. \)
\( D_{out} = (D_2, D_3, D_0) \)
and
\( B_{out} = (b_{2,0}, b_{2,1}) \)
\( \oplus \)
\( b_{2,2} \oplus b_{2,3} \)
\( b_{3,0} \),
\( b_{3,1} \),
\( b_{3,2} \)
\( \oplus \)
\( b_{3,3} \).

As the block cipher part of CSA hardly relates to our
work, we won’t introduce the details of the block cipher
part of CSA here, for more details on the structure of
the block cipher we refer the readers to the reference

3. Distinguishing Attack on CSA-SC

In this section we find related keys and initial value
of the CSA-SC. For any \((K, IV)\) pair there exists related
\((K', IV')\) pair that generate 1 bit shifted key stream, thus
we can lead to a distinguishing attack on SC part of
CSA.

As the 64 bit key are filled into the registers directly
and the structure of the encryption and the initialization
process are similar to each other, which make the
distinguishing attack to be possible. The following
notations are used throughout the paper:

- \( S(t) \): The internal state of CSA-SC at time \( t \).
- \( A(t) \): The internal state of register \( A \) at time \( t \), \( A(i, j, \oplus) \) (0 ≤
\( i \leq 9, \oplus \leq 3 \) ) represents the \( i \)th bit of the \( j \)th stage for
register \( A \) at time \( t \).
- \( B(t) \): The internal state of register \( B \) at time \( t \), \( B(i, j, \oplus) \) (0 ≤
\( i \leq 9, \oplus \leq 3 \) ) represents the \( i \)th bit of the \( j \)th stage for
register \( B \) at time \( t \).
- \( X(t), Y(t), Z(t), E(t), F(t), D(t) \): The internal state of registers
\( X, Y, Z, E, F, D \) at time \( t \), \( X(i), Y(i), Z(i), E(i), F(i), D(i) \) (0 ≤ \( i \)
≤ 3) represent the \( i \)th bit of corresponding registers at
\( t \).
- \( p(t), q(t), c(t) \): The internal state of registers \( p, q, c \) at
time \( t \).

Before the key initialization process, \( A(0) \) and \( B(0) \) are
initialized with the 64 bit key as follows:

\[
A_{i,j}(0) = \begin{cases} 
1 & \text{if } (i \leq 7) \\
0 & \text{else} 
\end{cases}
\]

\[
B_{i,j}(0) = \begin{cases} 
1 & \text{if } (i \leq 7) \\
0 & \text{else} 
\end{cases}
\]

Among which \( k \) denotes the \( i \)th bit of \( K \), other registers
are set to zero.

To lead a slide resynchronization attack we should calculate
all the state vectors of registers at different times. Firstly
calculate state vectors \( A(t) \) and \( B(t) \) by
using key initialization algorithm.

![Figure 3. One step transformation of register A and B.](image)

Among Figure 3:

\((k'_3, k'_2, k'_1, k'_0) = D_{out}(0) \oplus X_{out}(0) \oplus T_{out} \oplus A(0') \oplus B(0) \oplus B_{out}(0)\)

We denote the related key \( K' \) as follows:

\(k'_3, k'_2, k'_1, k'_0, k_1, k_2, k_3, \ldots, k_{24}, k_{25}, k_{26}, k_{27}, k_{32}, k_{33}, k_{34}, k_{35}, \ldots, k_{68}, k_{69}, k_{70}, k_{71}, k_{72}, k_{73}, k_{74}, k_{75}, k_{76}, k_{77}, k_{78}, k_{79})\)

If the following conditions occur:

- Condition 1: \((k'_2, k'_3, k'_4, k'_5) = (0, 0, 0, 0)\).
- Condition 2: \(q(0) = 0\).
- Condition 3: \(IV(0) = (a, a, a, a, a, a, a, a, a, a, a, a, a)\).
- Condition 4: \(K' = (k'_1, k'_2, k'_3, k'_4, k'_5, k'_6, k'_7, k'_8, k'_9)\).

For condition 1, as we don’t know whether the \((k'_2, k'_3, k'_4, k'_5, k'_6, k'_7, k'_8, k'_9)\) = \((0, 0, 0, 0, 0, 0, 0, 0)\),
we can realize this by trying all the related keys \( \Delta K \), \( \Delta K(28, 29, 30, 31, 32, 33, 34, 35) \in GF(2^8) \),
the difference of other bits are all zero, there must be a certain \( \Delta K \) which
make condition 1 occur.

For condition 2, if \( q(0) = 0 \), the register \( E \) and \( F \) don’t
update for the moment and are kept zero for \( f = 0 \),
the probability of \( q(0) = 0 \) is 1/2. As \( q(0) = \frac{1}{2} \),
\( k_{12} = k_{10} \oplus k_{12} \), if we try all the related keys \( \Delta K \),
\( \Delta K(12) \in GF(2^8) \), the difference of other bits are all zero, it
can be easily proven that the condition 2 must within.

For condition 3, the \( IV \) can be chosen as \( \frac{a(a, a, a, a, a, a, a, a, a, a, a, a, a, a, a, a)}{a} \in GF(2^8) \),
then the even cycle and odd cycle share the same \( T' \) and \( T \).

As for condition 4, \( D_{out}(1) \) can be regarded as a
random vector, to guarantee the probability, we can
choose any of the other bits except these positions:

\( k_{12}, k_{29}, k_{29}, k_{30}, k_{31}, k_{60}, k_{61}, k_{62}, k_{63} \)
It can be easily proven that the probability of \( \mathcal{D}^{31}_{2} = 1 \) is about 0.98 if we use \( 2^{6} \) related key \( (\bar{u} - u - 2^{-15})^{2} \approx 0.98 \) To sum up, the total computational complexity of all the conditions is \( 2^{15} \) pairs of related keys.

It’s easy to see that, for \( K \), the internal states of the cipher during the initialization process are denoted as \( S^{(0)}, S^{(1)}, S^{(2)}, \ldots, S^{(31)} \) for \( K' \), the corresponding states are denoted as \( S'^{(0)}, S'^{(1)}, S'^{(2)}, \ldots, S'^{(31)} \), if the four conditions above appear, the following equations during the initialization process are kept:

\[
S^{(0)} = S^{(1)} = S^{(2)} = S^{(3)} = \ldots = S^{(30)} = S^{(31)}
\]

During the key generating process, the following relations are kept:

\[
S^{(31)} = S^{(32)}, S^{(32)} = S^{(33)}, S^{(33)} = S^{(34)} = \ldots
\]

Hence, for \( K \), CSA-SC generates 1 bit shifted of the keystream generated for \( K' \). Thus, the key stream can be distinguished from a purely random sequence with computational complexity of \( O(2^{15}) \).

Further, if the 1 bit shifted of the keystream generated, according to the \( \Delta K \) for condition 1, we can determine eight key bits \( (k_{28}, k_{29}, k_{30}, k_{31}, k_{40}, k_{61}, k_{62}, k_{63}) \).

As for condition 2, \( k_{39} = 0 \), we can determine \( k_{12} \) according to the equation \( k_{29} \oplus k_{16} = 0 \) and \( \Delta K \) for condition 2. Hence, we have known 9 key bits which will decrease the exhaustive search complexity from \( O(2^{64}) \) to \( O(2^{55}) \). Last of all, we want to stress that as the residue is directly XORed to the key stream of the SC, we can get the key stream generated by the SC easily, which makes our attack can be applied to the whole algorithm.

4. Remarks on Success Rate

According to the result in section 3, if we trying \( 2^{15} \) pairs of \( K \) and \( K' \), the 1 bit shifted keystream can occur with probability 0.98 and this probability can be increased to 0.9997 when trying \( 2^{16} \) pairs of \( K \) and \( K' \). There is also another instance which may misjudge the distinguisher of our attack. When the key candidate can’t satisfy the several conditions above, the internal states after 32 rounds of initialization can be viewed as random, so \( S^{(31)} \) is equal to \( S^{(32)} \) with probability of \( 2^{-89} \), the probability is so small that it hardly affect out result.

On the other hand, we should get enough key stream bits to distinguish the shifted key stream from the random case, this can be accessed from the residue bits. If we get \( n \) residue bits, the success rate can achieve \( 1 - 2^{-n} \), when \( n = 10 \), the success rate can achieve 0.999 and the key stream can be easily got from one residue of the cipher (Less than 64 bits). So, the success rate of our attack can be very close to 1.

5. Conclusions

Although, the algorithm CSA only has 64 bit key size, for many TV stations only 48 bit of entropy are used [9]. However, there haven’t any effective crypto results which can break the algorithm totally up to now. The structure of CSA and the design rationale are well worth studying which may improve the design of the symmetric encryption algorithms significantly. As the initialization process and the key stream generating process are similar, based on the thought of slide resynchronization attack we propose a distinguishing attack which can distinguish the key stream of the SC from a purely random sequence with computational complexity of \( O(2^{15}) \). At the same time, we can recover 9 bits of the key accordingly, which makes the exhaustive search complexity reduce to \( O(2^{55}) \). Our attack provides a clue to recover several key bits of CSA, which may be combined with other attacks to reduce the computational complexity further. The ability of CSA to resist other cryptanalysis is further to be studied.

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References


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