Edge Detection Based on the Newton Interpolation's Fractional Differentiation

Chaobang Gao[1, 2], Jiliu Zhou[2, 3], and Weihua Zhang[3]
[1]College of Information Science and Technology, Chengdu University, China
[2]Key Laboratory of Pattern Recognition and Intelligent Information Processing, Chengdu University, China
[3]School of Computer Science, Sichuan University, China

Abstract: *In this paper, according to the development of the fractional differentiation and its applications in the modern signal processing, we improve the numerical calculation of fractional differentiation by Newton interpolation equation, and propose a new mask, the Newton Interpolation's Fractional Differentiation (NIFD). Then, we apply this new mask to image edge detection and can obtain the better edge information image. In order to get continuous and thin edges, we synthesize a new gradient and adopt the non-maxima suppression method. For a comparison, we consider the edge map yielded by the sobel operator and canny operator. By contrast, we discover that the edge image obtained by NIFD operator is better than those of sobel and canny operators, and specially for a noisy image, NIFD operator has the best anti-noise ability.*

Keywords: *NIFD operator, edge detection, newton interpolation, fractional differentiation.*

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1. Introduction

Fractional differentiation [10], also called non-integer differentiation, is not a new concept: it dates back to Cauchy, Riemann, Liouville and Letnikov in the 19th century. Since then, several theoretical physicists and mathematicians have studied fractional differential equations, especially fractional-order linear differential equations. In comparison with integer-order differentiation, the fractional differentiation of direct current or low-frequency signal is often nonzero. Fractional differential processing is not only nonlinearly keeping signal's low-frequency and direct current components, but also nonlinearly enhancing the signal's high-frequency and middle-frequency components. Based on the special characteristics of fractional differentiation in the last two decades, fractional differentiation has played a very important role in various physical sciences fields, such as mechanics, electricity, chemistry, biology, economics, time and frequency domains system identification, notably control theory, and robotics. Recently, fractal theory is already used in fractal image processing [3, 4, 5, 6, 9, 12, 13].

In image processing, edge detection often makes use of integer-order differentiation operators, especially order 1 used by the gradient and order 2 by the Laplacian [1, 2, 7, 8, 14]. In [4, 5, 6, 9, 11, 12], the principles of non-integer order differentiation operators in edge detection is introduced. This paper demonstrates with details how using an edge detector Newton Interpolation's Fractional Differentiation (NIFD) can improve the criterions of thin detection and immunity to noise, which can be interpreted in term of robustness to noise in general.

Based on the existed theories of fractional calculus and their specific applications in digital image processing, we improve the numerical calculation of fractional differentiation by Newton interpolation equation and propose a new operator (NIFD operator) that can improve the criterions of thin detection and immunity to noise.

2. Fractional Differentiation Review

Differential operation is a basic type of mathematical calculations, it is widely used in the fields of signal analysis and processing. Recently, many researches have been reported about fractional calculus and its applications, and it has become more and more important in foundational research and engineering application. From a theoretical point of view, fractional differentiation extends the order of signal processing from integer-order to any order, which is an extension of information processing methods and means [3, 4, 5, 6, 9, 12, 13].

Grümwald-Letnikov definition of fractional calculus originates from the classical definition of integer-order differentiation for continuous function, which is deduced by generalizing differential order from integer to fraction [10]. Assume that $\forall v \in R$ (R represents the real set, $[v]$ is its integral part), the signal $F(t) \in [a,t]$, $a<t$, $a \in R$, $t \in R$, has m($m \in Z$, Z represents the integer set) order continuous

differentiation. When $v>0$, m is no less than $[v]$, so v order differentiation could be expressed in the form [10]:

$$ {}_a^G D_t^v F(t) = \lim_{\substack{h\to 0 \\ nh=t-a}} \sum_{m=0}^{n-1} \frac{(-1)^m}{h^v} \frac{\Gamma(v+1)}{\Gamma(m+1)\Gamma(v-m+1)} F(t-mh) \tag{1} $$

Equation 1 is also written as $\frac{d^v F}{dt^v}$, and the difference of fractional differentiation equation 1 is expressed as [4, 5, 6, 12]:

$$ \begin{aligned} \frac{d^v F}{dt^v} &= F(t)+(-v)F(t-1)+\frac{-v(-v+1)}{2}F(t-2) \\ &+ \frac{-v(-v+1)(-v+2)}{6}F(t-3)+\cdots \\ &+ \frac{\Gamma(n-v)}{\Gamma(n+1)\Gamma(-v)}F(t-n)+\cdots \end{aligned} \tag{2} $$

Similarly, for the signal $F(x, y)$, the v order differentiation could be expressed:

$$ \frac{\partial^v F}{\partial x^v} = \lim_{h\to 0} \sum_{m=0}^{\left[\frac{t-a}{h}\right]} \frac{(-1)^m}{h^v} \frac{\Gamma(v+1)}{\Gamma(m+1)\Gamma(v-m+1)} F(x-mh, y) \tag{3} $$

$$ \frac{\partial^v F}{\partial y^v} = \lim_{h\to 0} \sum_{m=0}^{\left[\frac{t-a}{h}\right]} \frac{(-1)^m}{h^v} \frac{\Gamma(v+1)}{\Gamma(m+1)\Gamma(v-m+1)} F(x, y-mh) \tag{4} $$

The differences of fractional partial differentiation respectively are expressed as [4, 5, 6, 12]:

$$ \begin{aligned} \frac{\partial^v F}{\partial x^v} &= F(x,y)+(-v)F(x-1,y)+\frac{-v(-v+1)}{2}F(x-2,y) \\ &+ \frac{-v(-v+1)(-v+2)}{6}F(x-3,y)+\cdots \\ &+ \frac{\Gamma(n-v)}{\Gamma(n+1)\Gamma(-v)}F(x-n,y)+\cdots \end{aligned} \tag{5} $$

$$ \begin{aligned} \frac{\partial^v F}{\partial y^v} &= F(x,y)+(-v)F(x,y-1)+\frac{-v(-v+1)}{2}F(x,y-2) \\ &+ \frac{-v(-v+1)(-v+2)}{6}F(x,y-3)+\cdots \\ &+ \frac{\Gamma(n-v)}{\Gamma(n+1)\Gamma(-v)}F(x,y-n)+\cdots \end{aligned} \tag{6} $$

According to equations 5 and 6, among the n non-zero coefficients, only the coefficient of the first term is the constant 1, the other $n-1$ non-zero coefficients are functions with respect to the fractional order v. We can prove the sum of n non-zero coefficients is non-zero [5, 12]. And it is one of the significant differences in the characteristics between the fractional differentiation and the integer-order differentiation.

3. Theory of NIFD

To make the fractional differential operator more precise, we can improve fractional differentiation algorithm. Next, the points $F(t - mh)$, $m = 0, 1, 2, ..., n$ in equation 2 are viewed as nodes. To express any point between $t - mh - h$ and $t-mh + h$, let $\xi = t - mh + \frac{v}{2}h$.

When $v \in [-2, 2]$, $\xi \in [t - mh - h, t-mh + h]$. Thus, for any three nodes $F(t - mh - h)$, $F(t - mh)$, $F(t - mh + h)$, using Newton interpolation equation, one has interpolation expression of the signal function $F(t)$ in equation 2:

$$ \begin{aligned} F(\xi) &= F[t-mh-h] + F[t-mh-h, t-mh] \\ &\times (\xi-(t-mh-h)) \\ &+ F[t-mh-h, t-mh, t-mh+h] \\ &\times (\xi-(t-mh-h))(\xi-(t-mh)) \end{aligned} \tag{7} $$

Where

$$ F[t-mh-h] = F(t-mh-h) \tag{8} $$

$$ F[t-mh-h, t-mh] = \frac{F(t-mh)-F(t-mh-h)}{h} \tag{9} $$

$$ \begin{aligned} & F[t-mh-h, t-mh, t-mh+h] \\ &= \frac{F[t-mh, t-mh+h]-F[t-mh-h, t-mh]}{2h} \\ &= \frac{\frac{F(t-mh+h)-F(t-mh)}{h}-\frac{F(t-mh)-F(t-mh-h)}{h}}{2h} \\ &= \frac{F(t-mh+h)-2F(t-mh)+F(t-mh-h)}{2h^2} \end{aligned} \tag{10} $$

Taking equations 8 to 10 into equation 7, we have:

$$ \begin{aligned} F(\xi) &= F(t-mh-h)+\frac{F(t-mh)-F(t-mh-h)}{h} \\ &\times (\xi-(t-mh-h)) \\ &+ \frac{F(t-mh+h)-2F(t-mh)+F(t-mh-h)}{2h^2} \\ &\times (\xi-(t-mh-h))(\xi-(t-mh)) \end{aligned} \tag{11} $$

Since $\xi=t-mh + \frac{v}{2}h$, equation 11 becomes:

$$ \begin{aligned} F(\xi) &= F(t-mh-h)+\frac{F(t-mh)-F(t-mh-h)}{h} \\ &\times \left(t-mh+\frac{v}{2}h-(t-mh-h)\right) \\ &+ \frac{F(t-mh+h)-2F(t-mh)+F(t-mh-h)}{2h^2} \\ &\times \left(t-mh+\frac{v}{2}h-(t-mh-h)\right)\left(t-mh+\frac{v}{2}h-(t-mh)\right) \\ &= F(t-mh-h) + \left(\frac{v}{2}+1\right)\left(F(t-mh)-F(t-mh-h)\right) \\ &+ \frac{v(v+2)}{8}\left(F(t-mh+h) - 2F(t-mh)+F(t-mh-h)\right) \\ &= \left[1-\left(\frac{v}{2}+1\right)+\frac{1}{8}v(v+2)\right]F(t-mh-h) \\ &+ \left[\left(\frac{v}{2}+1\right)-\frac{v(v+2)}{4}\right]F(t-mh) + \left[\frac{1}{8}v(v+2)\right]F(t-mh+h) \\ &= \left(\frac{1}{8}v^2-\frac{1}{4}v\right)F(t-mh-h) + \left(1-\frac{v^2}{4}\right)F(t-mh) \\ &+ \left(\frac{1}{8}v^2+\frac{1}{4}v\right)F(t-mh+h) \end{aligned} \tag{12} $$

Compare with $F(t)$ in equations 2 and 12 has introduced the signal value $F(\xi)$ on non-node. And the non-node signal value $F(\xi)$ is a linear combination of the nodes $F(t - mh - h)$, $F(t - mh)$, and $F(t - mh + h)$, which implies that $F(\xi)$ contains the more information in its neighborhood. As we known, the processed object of computer or digital filter is the limit number, the biggest variable of gray-level of digital image signal is also limited, and the shortest changing distance of gray-level is one pixel, that is $h=1$. Then $F(t)$ is replaced by $F(\xi)$ and taking equation 12 into equation 2, we can obtain the approximation of equation 2 as follows:

$$
\begin{aligned}
\frac{d^vF(t)}{dt^v} &\approx \sum_{m=0}^{n-1}\frac{(-1)^m}{h^v}\frac{\Gamma(v+1)}{\Gamma(m+1)\Gamma(v-m+1)}\left[\left(\frac{1}{8}v^2-\frac{1}{4}v\right)\right.\\
&\quad\times F(t-mh-h)+\left(1-\frac{v^2}{4}\right)F(t-mh)+\left.\left(\frac{v^2}{8}+\frac{v}{4}\right)F(t-mh+h)\right]\\
&=\left[\left(\frac{v^2}{8}-\frac{v}{4}\right)F(t-1)+\left(1-\frac{v^2}{4}\right)F(t)+\left(\frac{v^2}{8}+\frac{v}{4}\right)F(t+1)\right]\\
&\quad-v\left[\left(\frac{v^2}{8}-\frac{v}{4}\right)F(t-2)+\left(1-\frac{v^2}{4}\right)F(t-1)+\left(\frac{v^2}{8}+\frac{v}{4}\right)F(t)\right]\\
&\quad+\frac{-v(-v+1)}{2}\left[\left(\frac{v^2}{8}-\frac{v}{4}\right)F(t-3)+\left(1-\frac{v^2}{4}\right)F(t-2)\right.\\
&\quad+\left.\left(\frac{1}{8}v^2+\frac{1}{4}v\right)F(t-1)\right]+\cdots+\frac{\Gamma(v+1)}{\Gamma(m+1)\Gamma(v-m+1)}\\
&\quad\times\left[\left(\frac{v^2}{8}-\frac{v}{4}\right)F(t-m-1)+\left(1-\frac{v^2}{4}\right)F(t-m)\right.\\
&\quad+\left.\left(\frac{v^2}{8}+\frac{v}{4}\right)F(t-m+1)\right]+\cdots\\
&=\left(\frac{v^2}{8}+\frac{v}{4}\right)F(t+1)+\left[\left(1-\frac{v^2}{4}\right)-v\left(\frac{v^2}{8}+\frac{v}{4}\right)\right]F(t)\\
&\quad+\left[\left(\frac{v^2}{8}-\frac{v}{4}\right)-v\left(1-\frac{v^2}{4}\right)+\frac{-v(-v+1)}{2}\left(\frac{1}{8}v^2+\frac{1}{4}v\right)\right]F(t-1)\\
&\quad+\left[-v\left(\frac{v^2}{8}-\frac{v}{4}\right)+\frac{-v(-v+1)}{2}\left(1-\frac{v^2}{4}\right)\right.\\
&\quad+\left.\frac{-v(-v+1)(-v+2)}{6}\left(\frac{v^2}{8}+\frac{v}{4}\right)\right]F(t-2)\\
&\quad+\left[\frac{-v(-v+1)}{2}\left(\frac{v^2}{8}-\frac{v}{4}\right)+\frac{-v(-v+1)(-v+2)}{6}\left(1-\frac{v^2}{4}\right)\right.\\
&\quad+\left.\frac{-v(-v+1)(-v+2)(-v+3)}{24}\left(\frac{v^2}{8}+\frac{v}{4}\right)\right]F(t-3)\\
&\quad+\cdots+\left[\left(\frac{1}{8}v^2-\frac{1}{4}v\right)\frac{\Gamma(n-v-1)}{\Gamma(n)\Gamma(-v)}+\left(1-\frac{v^2}{4}\right)\frac{\Gamma(n-v)}{\Gamma(n+1)\Gamma(-v)}\right.\\
&\quad+\left.\left(\frac{1}{8}v^2+\frac{1}{4}v\right)\frac{\Gamma(n-v+1)}{\Gamma(n+2)\Gamma(-v)}\right]F(t-n)+\cdots\\
&=\left(\frac{v^2}{8}+\frac{v}{4}\right)F(t+1)+\left(-\frac{v^3}{8}-\frac{v^2}{2}+1\right)F(t)\\
&\quad+\left(\frac{v^4}{16}+\frac{5v^3}{16}-\frac{5v}{4}\right)F(t-1)\\
&\quad+\left(\frac{-v^5}{48}-\frac{5v^4}{48}+\frac{v^3}{12}+\frac{3v^2}{4}-\frac{v}{2}\right)F(t-2)\\
&\quad+\left(\frac{v^6}{192}+\frac{v^5}{48}-\frac{13v^4}{192}-\frac{3v^3}{16}+\frac{9v^2}{16}-\frac{v}{3}\right)F(t-3)\\
&\quad+\cdots+\left[\left(\frac{v^2}{8}-\frac{v}{4}\right)\frac{\Gamma(n-v-1)}{\Gamma(n)\Gamma(-v)}+\left(1-\frac{v^2}{4}\right)\frac{\Gamma(n-v)}{\Gamma(n+1)\Gamma(-v)}\right.\\
&\quad+\left.\left(\frac{v^2}{8}+\frac{v}{4}\right)\frac{\Gamma(n-v+1)}{\Gamma(n+2)\Gamma(-v)}\right]F(t-n)+\cdots
\end{aligned}
\tag{13}
$$

Equation 13 is called the NIFD of $F(t)$. Indeed, the expression can only get the approximated value due it simplifies fractional differentiation to multiplication and add. Similarly, we choose the top $n+2$ terms as the fractional differentiation approximation of $F(t)$. Let:

$$
\begin{cases}
a_{-1}=\dfrac{1}{8}v^2+\dfrac{1}{4}v\\
a_0=-\dfrac{v^3}{8}-\dfrac{v^2}{2}+1\\
a_1=\dfrac{v^4}{16}+\dfrac{5v^3}{16}-\dfrac{5v}{4}\\
a_2=-\dfrac{v^5}{48}-\dfrac{5v^4}{48}+\dfrac{v^3}{12}+\dfrac{3v^2}{4}-\dfrac{v}{2}\\
a_3=\dfrac{v^6}{192}+\dfrac{v^5}{48}-\dfrac{13v^4}{192}-\dfrac{3v^3}{16}+\dfrac{9v^2}{16}-\dfrac{v}{3}\\
\cdots\\
a_n=\left(\dfrac{1}{8}v^2-\dfrac{1}{4}v\right)\dfrac{\Gamma(n-v-1)}{\Gamma(n)\Gamma(-v)}+\left(1-\dfrac{v^2}{4}\right)\dfrac{\Gamma(n-v)}{\Gamma(n+1)\Gamma(-v)}\\
\qquad+\left(\dfrac{1}{8}v^2+\dfrac{1}{4}v\right)\dfrac{\Gamma(n-v+1)}{\Gamma(n+2)\Gamma(-v)}
\end{cases}
\tag{14}
$$

Thus,

$$
\begin{aligned}
\frac{d^vF(t)}{dt^v} &\approx a_{-1}F(t+1)+a_0F(t)+a_1F(t-1)\\
&\quad+\cdots+a_nF(t-n)
\end{aligned}
\tag{15}
$$

Similarly, for the signal $F(x,y)$, from equation 13, the approximate backward differences of fractional partial differentiation respectively on negative x-coordinate and y-coordinate, are expressed as:

$$
\begin{aligned}
\frac{\partial^vF}{\partial x^v} &= \left(\frac{v^2}{8}+\frac{v}{4}\right)F(x+1,y)+\left(-\frac{v^3}{8}-\frac{v^2}{2}+1\right)F(x,y)\\
&\quad+\left(\frac{v^4}{16}+\frac{5v^3}{16}-\frac{5v}{4}\right)F(x-1,y)+\left(\frac{-v^5}{48}-\frac{5v^4}{48}+\frac{v^3}{12}+\frac{3v^2}{4}-\frac{v}{2}\right)F(x-2,y)\\
&\quad+\left(\frac{v^6}{192}+\frac{v^5}{48}-\frac{13v^4}{192}-\frac{3v^3}{16}+\frac{9v^2}{16}-\frac{v}{3}\right)F(x-3,y)+\cdots+\left[\left(\frac{v^2}{8}-\frac{v}{4}\right)\frac{\Gamma(n-v-1)}{\Gamma(n)\Gamma(-v)}\right.\\
&\quad+\left.\left(1-\frac{v^2}{4}\right)\frac{\Gamma(n-v)}{\Gamma(n+1)\Gamma(-v)}+\left(\frac{v^2}{8}+\frac{v}{4}\right)\frac{\Gamma(n-v+1)}{\Gamma(n+2)\Gamma(-v)}\right]F(x-n,y)+\cdots
\end{aligned}
\tag{16}
$$

$$
\begin{aligned}
\frac{\partial^vF}{\partial y^v} &= \left(\frac{v^2}{8}+\frac{v}{4}\right)F(x,y+1)+\left(-\frac{v^3}{8}-\frac{v^2}{2}+1\right)F(x,y)\\
&\quad+\left(\frac{v^4}{16}+\frac{5v^3}{16}-\frac{5v}{4}\right)F(x,y-1)+\left(\frac{-v^5}{48}-\frac{5v^4}{48}+\frac{v^3}{12}+\frac{3v^2}{4}-\frac{v}{2}\right)F(x,y-2)\\
&\quad+\left(\frac{v^6}{192}+\frac{v^5}{48}-\frac{13v^4}{192}-\frac{3v^3}{16}+\frac{9v^2}{16}-\frac{v}{3}\right)F(x,y-3)+\cdots+\left[\left(\frac{v^2}{8}-\frac{v}{4}\right)\frac{\Gamma(n-v-1)}{\Gamma(n)\Gamma(-v)}\right.\\
&\quad+\left.\left(1-\frac{v^2}{4}\right)\frac{\Gamma(n-v)}{\Gamma(n+1)\Gamma(-v)}+\left(\frac{v^2}{8}+\frac{v}{4}\right)\frac{\Gamma(n-v+1)}{\Gamma(n+2)\Gamma(-v)}\right]F(x,y-n)+\cdots
\end{aligned}
\tag{17}
$$

To obtain new mask along eight symmetric directions and make them have anti-rotation capability, eight new masks which are respectively along the directions of negative x-coordinate, negative y-coordinate, positive x-coordinate, positive y-coordinate, left downward diagonal, right upward diagonal, left upward diagonal, and right downward diagonal are implemented, which is noted as NIFD operators and the operator along every direction is written as NIFD_i, $i = 0, 1, \ldots, 7$, as shown in Figure 1.

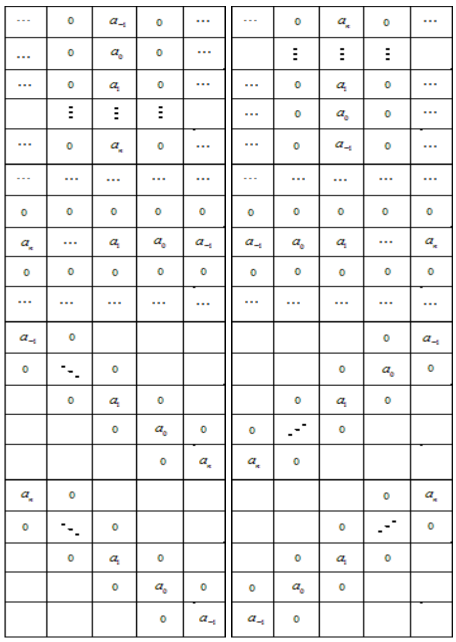


Figure 1. NIFD mask.

4. Experiment Results and Analysis

According to Figure 1, we adopt 5×5 mask. For an image, first do Gauss filter, and then calculate the fractional differentiation by NIFD mask, thus eight edge information images are obtained by $NIFD_i$, $i=0,1,\ldots,7$ along eight directions respectively. Some results are shown in Figure 2.

a) Original.

b) Edge information by $NIFD_0$.

c) Edge information by $NIFD_1$.

d) Edge information by $NIFD_2$.

e) Edge information by $NIFD_3$.

f) Edge information by $NIFD_4$.

g) Edge information by $NIFD_5$.

h) Edge information by $NIFD_6$.

i) Edge information by $NIFD_7$.

Figure 2. The edge information image of 0.8 order by $NIFD_i$, $i = 0, 1, \ldots, 7$.

From the eight edge information images obtained by $NIFD_i$, $i = 0, 1, \ldots, 7$, we can project them to two directions (i.e., linear combination):

$$\begin{aligned} d_x f(x,y) = {} & NIFD_2 f(x,y) - NIFD_3 f(x,y) \\ & + \frac{\sqrt{2}}{2}\big(NIFD_4 f(x,y) - NIFD_5 f(x,y) \\ & + NIFD_6 f(x,y) - NIFD_7 f(x,y)\big) \end{aligned} \tag{18}$$

$$\begin{aligned} d_y f(x,y) = {} & NIFD_0 f(x,y) - NIFD_1 f(x,y) \\ & + \frac{\sqrt{2}}{2}\big(NIFD_4 f(x,y) - NIFD_6 f(x,y) \\ & + NIFD_5 f(x,y) - NIFD_7 f(x,y)\big) \end{aligned} \tag{19}$$

Then, we synthesize a new gradient for an image *f(x,y)*:

$$\begin{aligned} gradf(x,y) = {} & d_x f(x,y) i + d_y f(x,y) j \\ = {} & \big[NIFD_2 f(x,y) - NIFD_3 f(x,y) \\ & + \frac{\sqrt{2}}{2}\big(NIFD_4 f(x,y) - NIFD_5 f(x,y) \\ & + NIFD_6 f(x,y) - NIFD_7 f(x,y)\big)\big] i \\ & + \big[NIFD_0 f(x,y) - NIFD_1 f(x,y) \\ & + \frac{\sqrt{2}}{2}\big(NIFD_4 f(x,y) - NIFD_6 f(x,y) \\ & + NIFD_5 f(x,y) - NIFD_7 f(x,y)\big)\big] j \end{aligned} \tag{20}$$

The norm of the synthesized gradient is:

$$\|gradf(x,y)\| = \sqrt{d_x^2 + d_y^2}$$

In order to get continuous and thin edges, next we need to track and connect the edges of image based on the norm image of the gradient. The idea is the same as the non_maxima suppression method [2]. For a comparison, we considered the edge map yielded by sobel and canny operators. Some results are shown in Figure 3.

a) Original.

b) NIFD v = 0.1.

c) NIFD v = 0.3.

d) NIFD v = 0.5.

e) NIFD v = 0.7.

f) NIFD v = 0.9.

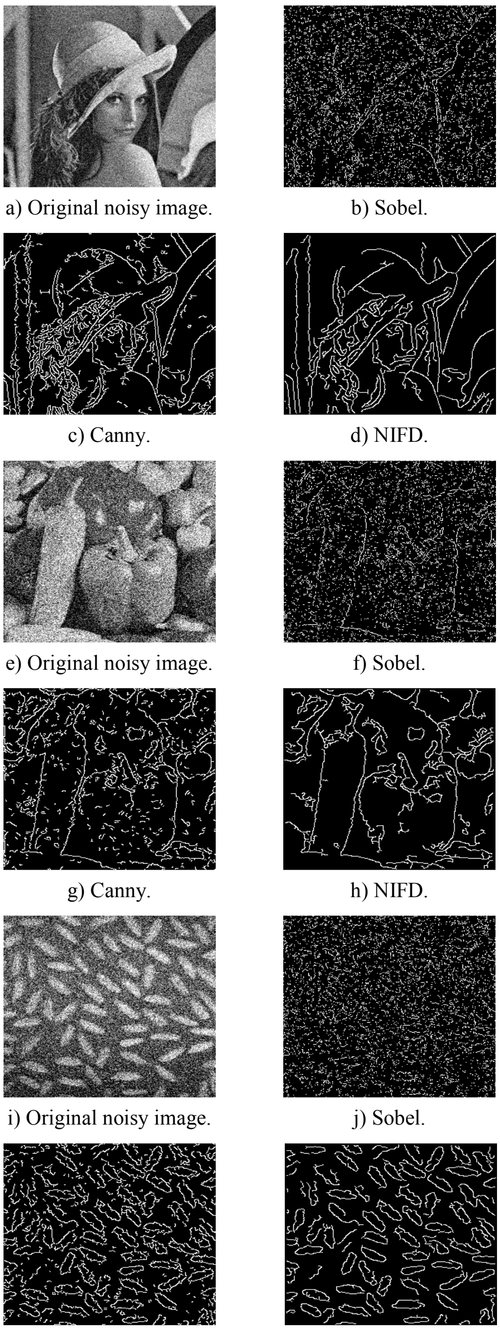
g) Sobel.

h) Canny.

Figure 3. Comparison of edge images.

Observing carefully Figure 3, we see that NIFD operator and canny operator have stronger edge search capability and more complete edge, but sobel operator can not. In addition, NIFD operator and canny operator almost completely maintain contour information of the original image, but only NIFD operator can weaken false negatives in the textured regions.

It is known that accurate detection of edges in noisy data should comply with two possibly conflicting requirements. The edge detection process should avoid false edges produced by noise and ensure that actual edges are correctly detected. In order to validate the performance of NIFD operator, we generate some noisy images by adding zero-mean Gaussian noise with standard deviation 0.05, then detect edge of noisy images by sobel, Canny and NIFD operators. A part of results are shown in Figure 4.



a) Original noisy image. b) Sobel.
c) Canny. d) NIFD.
e) Original noisy image. f) Sobel.
g) Canny. h) NIFD.
i) Original noisy image. j) Sobel.
k) Canny. l) NIFD.

Figure 4. Comparison of edge images obtained by different operators for noisy images.

From Figure 4, we see that the better performance of NIFD operator is apparent. The edge curve obtained by the NIFD operator is always smoother than that of canny operator. For sobel operator, the edge map is noisy, and this effect is very annoying. And we can easily see that the edge map given by NIFD looks sharper than those yielded by sobel operator and canny operator. The lower resolution yielded by the sobel operator is clearly perceivable. The edge curves have not been detected. Although, canny operator can extract edge curves, the result is more noisy and the edge curves contain always some 'glitch'. Since canny operator tracks and connects the edges of image based on the norm image of gradient that is obtained by filtering along two directions. While NIFD operator is based on the norm image of gradient that is synthesized by filtering along eight directions such as equations 18 and 19, thus it has anti-rotation capability and anti-noise ability. Therefore, the NIFD operator offers the best result among them. The edge curves can satisfactorily been detected and be much less noisy by NIFD operator, and the image edges look very sharp. NIFD operator has stronger edge search capability and more complete edge, and may overcome the shortcomings of sobel and canny operators.

5. Conclusions

Fractional differentiation has played a very important role in digital image processing fields respectively and more and more researchers begin to study them. This paper intends to deduce a new operator, NIFD operator, which may be applied to edge detection. Experiments show that the NIFD operator has excellent edge detection capabilities and especially plays an important role in reducing the noise sensitivity.

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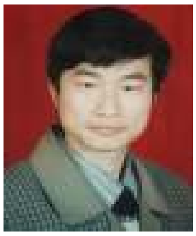
Chaobang Gao is a PhD and associate professor with the College of Computer Science and Technology, Chengdu University. His research interests cover digital image processing, artificial intelligence, biology feature recognition, the theory of fractional calculus and its application in signal processing. He studies currently the theory and application of fractional directional differentiation. He published or will publish more than 30 papers which involve fractional directional differentiation and quaternion fractional calculus.

Jiliu Zhou is currently a Professor with the School of Computer Science, Sichuan University. His research is mainly in the field of image processing, artificial intelligence, fractional differential application on the latest signal and image processing, and so on. He has published more than 100 papers, of which more than 80 papers are indexed by SCI, EI, or ISTP.



Weihua Zhang is currently a PhD with the School of Computer Science, Sichuan University. His research interests cover digital image processing, artificial intelligence, biology feature recognition, the theory of fractional calculus and its application in signal processing.