Using Quantum-Behaved Particle Swarm Optimization for Portfolio Selection Problem

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Abstract: *One of the popular methods for optimizing combinational problems such as portfolio selection problem is swarm-based methods. In this paper, we have proposed an approach based on Quantum-Behaved Particle Swarm Optimization (QPSO) for the portfolio selection problem. The Particle Swarm Optimization (PSO) is a well-known population-based swarm intelligence algorithm. QPSO is also proposed by combining the classical PSO philosophy and quantum mechanics to improve performance of PSO. Generally, investors, in portfolio selection, simultaneously consider such contradictory objectives as the rate of return, risk and liquidity. We employed QPSO model to select the best portfolio in 50 supreme tehran stock exchange companies in order to optimize the objectives of the rate of return, systematic and non-systematic risks, return skewness, liquidity and sharp ratio. Finally, the obtained results were compared with Markowitz`s classic and Genetic Algorithms (GA) models indicated that although return of the portfolio of QPSO model was less that that in Markowitz's classic model, the QPSO had basically some advantages in decreasing risk in the sense that it completely covers the rate of return and leads to better results and proposes more versatility portfolios in compared with the other models. Therefore, we could conclude that as far as selection of the best portfolio is concerned, QPSO model can lead to better results and may help the investors to make the best portfolio selection.*

Keywords: *Swarm algorithm, portfolio selection, GA, risk, return.*

Received January 2, 2010; accepted August 10, 2010

1. Introduction

Markowitz's work [17, 18] is considered as the most influential theory for portfolio selection. Since he proposed his distinguished work using quantitative methods, scholars began to research portfolio selection. Markowitz analyzed portfolios containing large numbers of securities. He considered returns of individual securities as random variables. The purpose of his analysis was to find portfolios that best meet the objectives of investors. Markowitz presented his famous mean-variance model through quantifying portfolio return as mean and calculating variance as risk. For a given specific return level, an optimal portfolio can be obtained when the variance of portfolio is minimized. While, for a given risk level, which the investor can bear, an optimal portfolio can be obtained when the expected return of portfolio is maximized.

Since the introduction of Markowitz's model, many efforts have been made to improve mean-variance models. Sharp [25] greatly simplified the number and the type of input data. Best [2], Merton [22], and Voros [32] contributed greatly to the efficient portfolio frontier. Based on the core of Markowitz's theory of applying mean and variance to describe return and risk, mean-variance models were also extended to discuss various issues in portfolio management. Huang and Litzenberger [12] researched portfolio selection theory when a riskless asset and short-selling of riskless asset are allowed. Konno and Suzuki [16] considered skewness in portfolio management.

These researches greatly developed Markowitz's mean-variance model and made great achievements in portfolio selection theory. The common assumptions in their models are that investors have enough historical data of securities and that the situation of asset markets in future can be correctly reflected by the asset data in the past. However, it is hard to always ensure such assumptions. Sometimes, for example, when new stocks are listed in the stock market, there is no past performance information for their securities; and for an ever-changing real asset market, the second assumption can hardly be ensured either.

Although, there are different methods to solve portfolio selection and other combinatorial optimization problems in financial and other applied sciences, use of Genetic Algorithm (GA) is promising approach to solve combinatorial optimization problems [3].

The GA, introduced by Holland in 1975, is a well-known efficient nonlinear search methodology in large spaces [10]. GA uses a population, which is simply a set of chromosomes, to search the solution space.

During each generation, three genetic operators are applied to the population: selection, crossover and mutation. Selection operator picks chromosomes in the population based on their fitness. Each pair of chromosomes or parents undergo crossover at random by exchanging their information with each other to generate new chromosome or offspring. The efficient set selection within a portfolio may be efficiently solved by using GA [28, 34, 35]. These GA based portfolio selection algorithms merely focus on standard deviation as an appropriate measure for non-systematic risk. This indicates that investors weigh the probabilities of negative returns equally against positive returns [6]. In addition, GA has also been used to measure portfolio volatility in relation to the stock market [24] to evidence that the technical trading is efficient in the Santa Fe artificial stock market [8] and to explore the relationship between the wealth dynamics and risk preferences in a multi-asset artificial stock market [7]. As using genetic algorithm is common and different articles were covered this problem, we just show the results of running this method in order to focus on proposed Quantum Behaved Particle Swarm Optimization algorithm (QPSO).

Another popular method for optimizing complex problems, which is kind of population-based algorithms, is to apply swarm algorithm because it can be implemented relatively easily and is applicable to a very wide range of problems. Swarm algorithm is a type of artificial algorithm based on the collective behaviour of decentralized, self-organized systems. One of the famous swarm based algorithms that is introduced so far is Particle Swarm Optimization (PSO). The PSO belongs to the class of direct search methods used to find an optimal solution to an objective function (aka fitness function) in a search space, which is first described in 1995 by Kennedy and Eberhart [15]. PSO is not a global convergence-guaranteed optimization algorithm, as van den Bergh has demonstrated [33]. Therefore, Sun *et al.* [29, 30] proposed a global convergence-guaranteed search technique, QPSO whose performance is superior to the PSO [11]. The QPSO is proposed by combining the classical PSO philosophy and quantum mechanics to improve performance of PSO.

In this paper, risk evaluations concentrate on estimating the distributions of financial return series. The main objective is to optimize the portfolio set based on appropriate threshold selection. Thus, we introduce a QPSO based portfolio volatility to forecast a model in order to select the best portfolio set and dynamically to estimate a suitable peak threshold for each asset in the portfolio, simultaneously. The proposed method is highly efficient and effective in providing near optimal solutions within a few minutes. The paper is organized as follows: In section 2, Markowitz models for portfolio selection are briefly reviewed and after the introduction of some necessary knowledge about PSO, A QPSO-Portfolio Selecting algorithm is designed to solve the portfolio selection problem in section 3 that is covered all aspects of research methodology. In section 4 results will test with different ratios and in the end of this part, numerical results are presented to show the potential applications of the different models. Conclusion remarks are finally given in section 5.

2. Definition and Literature

2.1. Portfolio Selection

First, let us remember the well-known Markowitz's mean-variance model [17, 18] for portfolio selection problem. Let N be the number of different assets, μ_i be the mean return of asset i, σ_{ij} be the covariance between the returns of assets i and j, and finally, λ [0, 1] be the risk aversion parameter. The decision variables x_i represent the proportion of capital to be invested in the asset i. using this notation, the standard Markowitz's mean-variance model for the portfolio selection problem would be:

$$minimize \quad \lambda\left[\sum_{i=1}^{N}\sum_{j=1}^{N} x_i \sigma_{ij} x_j\right] + (1-\lambda)\left[-\sum_{i=1}^{N} \mu_i x_i\right] \tag{1}$$

$$subject \quad to \quad \sum_{i=1}^{N} x_i = 1, \tag{2}$$

$$0 \le x_i \le 1, \; i = 1, \ldots, N. \tag{3}$$

The case with λ=0 represents maximizing of the portfolio mean return without considering the variance and the optimal solution will form only by the asset with the greatest mean return. The case with λ=1 represents minimizing of the total variance associated to the portfolio regardless of the mean return and the optimal solution will typically consist of several assets. Any value of λ inside the interval (0, 1) represents a tradeoff between the mean return and the variance, generating a solution between the two extremes (λ=0 and 1).

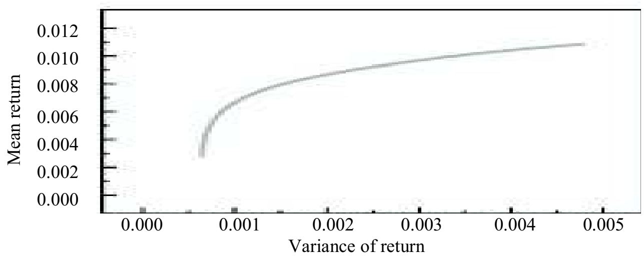

Figure 1. Standard efficient frontier corresponding to the smallest benchmark problem.

Since every solution satisfying all the constraints (feasible solution) corresponds with one of the possible portfolios, from now on, we will speak without distinguishing between the solutions for the above problem and portfolios. The portfolio selection problem is an instance of the family of multi-objective optimization problems. Therefore, first we have to find out adopt a definition for the concept of optimal solution. Here we will use the Pareto optimality definition [26].

"A feasible solution of the portfolio selection problem will be an optimal solution (or non-dominated solution) if there is not any other feasible solution improving one objective without making worse the other".

Usually a multi-objective optimization problem has different optimal solutions. The objective function values of all these non-dominated solution form what the so-called the "efficient frontier". For the problem defined in equations 1-3, the efficient frontier is an increasing curve that gives the best tradeoff between the mean return and variance (risk). Figure 1 shows an example of such a curve corresponding to the smallest benchmark problem described in section 4. This efficient frontier has been computed taking 2000 different values for the risk aversion parameter λ and solving exactly the corresponding portfolio selection problems. The objective function values of the resulting solutions give 2000 points that form the curve in Figure 1. We call this curve as the standard efficient frontier in order to distinguish it from the general efficient frontier, corresponding to the general mean-variance portfolio selection model which we will be describes in the coming paragraphs.

With the purpose of generalizing, the standard Markowitz's model to include cardinality and bounding constraints, the model formulation that can also be found in [4, 13, 26] will be applied. In addition to the previously defined variables, let K be the desired number of different assets in the portfolio with no null investment, ε_i and δ_i be the lower and upper bounds respectively for the proportion of capital to be invested in the asset i, with $0 \le \varepsilon_i \le \delta_i \le 1$. The additional decision variables z_i are 1 if asset i is included in the portfolio and 0 otherwise. The general mean-variance model for the portfolio selection problem is:

$$\text{minimize} \quad \lambda\left[\sum_{i=1}^{N}\sum_{j=1}^{N} x_i \sigma_{ij} x_j\right] + (1-\lambda)\left[-\sum_{i=1}^{N}\mu_i x_i\right] \quad (4)$$

$$\text{subject to} \quad \sum_{i=1}^{N} x_i = 1, \quad (5)$$

$$\sum_{i=1}^{N} z_i = K, \quad (6)$$

$$\varepsilon_i z_i \le x_i \le \delta_i z_i, \quad i = 1,\ldots,N, \quad (7)$$

$$z_i \in \{0,1\}, \quad i = 1,\ldots,N \quad (8)$$

This formulation is a mixed quadratic and integer-programming problem, for which no efficient algorithms exist. Another difference with the standard model is that, in the presence of cardinality and bounding constraints the resulting efficient frontier (which we are going to call it as general efficient frontier) can be quite different from the one obtained with the standard mean-variance model. In particular, the general efficient frontier may be discontinuous [4, 13].

2.2. A Quantum-Behaved Particle Swarm Optimization

The PSO algorithm, firstly proposed by Kennedy and Eberhart [15], is a population-based evolutionary search technique. It is underlying motivation for the development of PSO was social behavior of animals such as bird flocking, fish schooling, and animal herding and swarm theory. In PSO with M individuals, a potential solution to a problem is represented as a particle flying in D dimensional search space, with the position vector $X_i=(x_{i1}, x_{i2},\ldots,x_{iD})$ and velocity $V_i=(v_{i1},v_{i2},\ldots,v_{iD})$. Each particle records its best previous position (the position giving the best fitness value) as $pbest_i=(pbest_{i1}, pbest_{i2}, \ldots, pbest_{iD})$ called personal best position. At each iteration, each particle competes with the others in the neighborhood or in the whole population for the best particle (with best fitness value among neighborhood or the population) with best position $gbest_i=(gbest_{i1}, gbest_{i2}, \ldots, gbest_{iD})$ called global best position, and then makes stochastic adjustment according to the following evolution equations.

$$v_{id} = w.v_{id} + c_1.rand_1().(pbest_{id} - x_{id}) + c_2.rand_2().(gbest - x_{id}) \quad (9)$$

$$x_{id} = x_{id} + v_{id} \quad (10)$$

For i=1,2,…,M; d=1,2,…,D. In equation 9, c_1 and c_2 are positive constant; $rand_1()$ and $rand_2()$ are two random functions generating uniformly distributed random numbers within [0, 1]. Parameter w is the inertia weight introduced to accelerate the convergence speed of the PSO. At each iteration, the value of V_{id} is restricted in [-$Vmax$, $Vmax$].

PSO is not a global convergence-guaranteed optimization algorithm, as Van Den Bergh has demonstrated [33]. Therefore, Sun *et al.* [29, 30] proposed a global convergence-guaranteed search technique, algorithm QPSO, whose performance is superior to the PSO. In the quantum model of a PSO, the state of a particle is depicted by wave function $\psi(x, t)$, instead of position and velocity. The dynamic behaviour of the particle is widely different from that of the particle in traditional PSO systems in that the exact values of position and velocity cannot be determined simultaneously. We can only learn the probability of the particle's appearing in position x from probability density function $|\Psi(x,t)|^2$, the form of which depends on the potential field the particle lies in.

The particles move according to the following iterative equation [28, 29].

$$x_{id}(t+1) = \{g_{id} \pm \beta \mid mbest_d - x_{id}(t) \mid ln(1/u)\} \quad (11)$$

where

$$g_{id} = \varphi.pbest_{id} + (1-\varphi)gbest_d, \quad (12)$$

and

$$mbest_d = \sum_{i=1}^{M} pbest_{id} / M \quad (13)$$

mbest (mean best position or mainstream thought point) is defined as the mean value of all particles' the best position, φ and u are random number distributed uniformly on [0,1] respectively and m is the number of particles. $L=\beta. |mbest_d - x_{id}(t)|.1n(1/u)$ can be viewed as the strength of creativity or imagination because it characterizes the knowledge seeking scope of the particle, and therefore the larger the value L, the more likely the particle find out new knowledge. The parameter, β, called contraction-expansion coefficient, is the only parameter in QPSO algorithm. From the results of stochastic simulations, QPSO has relatively better performance by varying the value of β from 1.0 at the beginning of the search to 0.5 at the end of the search to balance the exploration and exploitation [31].

3. Research Methodology

3.1. The Objectives of Model

As already mentioned, the main objectives of our model for estimating the best results for the portfolio selection include.

3.1.1. Maximizing the Portfolio Return

Since investment is carried out for obtaining return and investors try to invest in a manner so as to be able to achieve the maximum level of return, accordingly the purpose of portfolio selection can be defined as: "Return of investment in stock including any type of cash within a given period of time along with the fluctuations of price during the period divided by the price of securities or assets at the time of buying" [1]. To calculate the rate of return the equation 14 is used:

$$max\, Z_1 = \sum_{i=1}^{n} x_i r_i \quad (14)$$

where, x_i is the proportion invested in various assets when the best trade-off is found and r_i is the expected rate of the return of assets.

3.1.2. Minimizing the Non-Systematic Risk

Since we have defined risk as the fluctuation of return, then the more limited the distribution of the return, the less amount of the risk we would expect. In practice, we can use "standard deviation of the rate of return" which shows the characteristics of probability distribution for measuring the amount of risk [18]. Regarding the fact that variance reveals the distribution of the data around the mean, minimizing of the significance of variance, as an objective, to decrease the fluctuation of a portfolio return can be stated as equation 15:

$$min Z_2 = \sum_{i=1}^{n} x_i^2 \delta_i^2 + \sum_{i=1}^{n} \sum_{j=1}^{n} x_i x_j \delta_{ij} \quad (15)$$

where, δ_i^2 and δ_{ij} are the variance and covariance of the excess returns, respectively.

3.1.3. Minimizing the Systematic Level of Risk

Beta as an indicator of sensitivity is a criterion for measuring the systematic level of risk, which measures part of the total risk that does not reduce as variety increases. Beta is a relative criterion of a given share in relation to the portfolio of the whole stock. The amount of beta can be calculated using the following equation [24]. The objective of the minimum systematic level of risk can also be defined as the equation 16 in order to minimize this type of risk:

$$min\, Z_3 = \sum_{i=1}^{n} x_i \beta_i \quad (16)$$

where, β_i is the systematic risk of assets.

3.1.4. Maximizing the Stock Return Skewness

Considering the fact that the investor looks for positive return and tries to select a stock whit a positive distribution of return, and since the companies under investigation had positive distribution of return skewness [16], the following objective was defined for selecting a portfolio with a positive distribution of return skewness:

$$MaxZ_4 = \sum_{i=1}^{n} S_{iii}^3 X_i^3 + 3\sum_{i=1}^{n}\left(\sum_{j=i}^{n} X_i^2 X_j S_{iij} + \sum_{j=1}^{i-1} X_i X_j^2 S_{ijj}\right) \quad i \neq j \quad (17)$$

where, S_{iii}^3 is the skewness and, S_{iij} and S_{ijj} are co-skewness of the excess returns.

3.1.5. Maximizing the Level of Portfolio Liquidity

The amount of liquidity stock reveals the potentiality of changing stock into other types of equities such as money. Since in emergency cases investors tend to sell their stock easily in order to calculate the risk of company liquidity, we can use the ratio of the number of days in which the company's stock was dealt over the number of the days in which the company was active in the market. Therefore, the objective of maximizing portfolio liquidity can be stated as:

$$max\, Z_5 = \sum_{i=1}^{n} x_i e_i \quad (18)$$

where, e_i is the liquidity of assets.

3.1.6. Maximizing the Sharp Rate in Portfolio

Sharp (1963) introduced the criterion of additional return to the risk as an indicator of developing portfolio equation 19. As already explained, additional return is the difference between the return without risk and the return of the stock and the investors' aim to invest in a way to achieve the maximum of this level and more return at the expense of risk they accept:

$$max\, Z_6 = \sum_{i=1}^{n} x_i S_i \qquad (19)$$

where, S_i is the liquidity of assets.

3.2. The Quantum-Behaved Particle Swarm Optimization for Solving Portfolio Selection Problem

In order to solve determine the level of investment in a portfolio with QPSO, we must determine an encoding of particle's position and a fitness function and then we must introduce an algorithm based on QPSO.

3.2.1. Particle's Position

A particle's position encoding is most important factor in QPSO that is affected on the size of the search space. The particle's position of the current problem has 50 genes. The decimal of each gene indicates a collection of answers related to the amount of investment in each company. Figure 2 shows the particle's position encoding.

Index :	1	2	…	50
The amount of investment:	0.1	0.23	…	0.89

Figure 2. A particle's position encoding.

3.2.2. Fitness Function

Another important factor is a fitness function. The complete and appropriate fitness function is showed in equation 20:

$$F(x) = \frac{n_1 - Z_1}{h_1} + \frac{Z_2 - p_2}{h_2} + \frac{Z_3 - p_3}{h_3} + \frac{n_4 - Z_4}{h_4} + \frac{n_5 - Z_5}{h_5} + \frac{n_6 - Z_6}{h_6} \qquad (20)$$

where Z_i, $n_i, (p_i)$ and h_i are respectively the objective functions, the fitness amounts (that described in section 4.1) and euclidean normalized index. In this paper, we must try to minimize fitness function.

3.2.3. The Proposed Algorithm

The following is the procedure of QPSO-Portfolio Selecting algorithm:

- *Step 1: Initialize the population by randomly generate the position vector X_i of each particle and set $pbest_i = X_i$.*
- *Step 2: Evaluate the fitness value of each particle by equation 20, update the personal best position (pbesti) and obtain the global best position (gbest) across the population.*
- *Step 3: If the stop criterion is met, go to step hg5; or else go to step 4.*
- *Step 4: Update the position vector of each particle according to equation 11 and go to step 2.*
- *Sep 5: Output the gbest as a solution for portfolio selection problem.*

There are two alternatives for stop criterion of the algorithm. One method is that the algorithm stops when the objective function value is less than a given threshold ε; the other is that it terminates after executing a pre-specified number of iterations.

4. Test of Results with Different Ratios (Running the Model)

As was mentioned, the objective was to select the best portfolio from among the 50 top companies of the Tehran Stock Exchange. Table 1 shows these companies along with the defined variables and the necessary parameters for applying the model.

As Table 1 shows the first column lists the companies in question, the second column shows the decided variables for each company and the third column is the monthly return for each company, which was calculated based on the mean of returns in the last 72 months ending in March 2008. To measure non-systematic risk in the fourth column, we used variance of return. The fifth column demonstrates systematic risk (beta sensitivity indicator) and the sixth column indicates the skewness return of stock within the investigated ranges (since the objective was to maximize the skewness of portfolio, in this study, the 50 top companies in the stock which had positive skewness indicator were investigated). The seventh column represents the liquidity of each company, and the last column indicates the sharp rate, demonstrates the additional return of the stock in relation to non-systematic risk. To measure the sharp ratio, free-risk return was considered as 15.5% (as the mean rate of the commercial securities).

Table 1. Coefficient of the stocks in the model.

Stocks	Variable	Rate of Return	Nonsystematic risk	Systematic risk	Skewness of return	Liquidity	Sharp ratio
Iran Khodro	X_1	3.35	167.7025	2.04	3.872	0.89	0.26
Iran Khodro Diesel	X_2	4.01	317.5524	2.73	3.872	0.85	0.22
Bank Eghtesade Novin	X_3	9.83	1323.504	3.18	3.872	0.83	0.27
…	…	…	…	…	…	…	…
Behran Oil	X_{49}	3.48	89.1136	0.47	3.872	0.8	0.37
Pars Oil	X_{50}	2.78	114.49	0.63	3.872	0.77	0.26

4.1. Limitation Levels and Fitness Amount

Reviewing similar cases and conducting interviews with stock specialists and practitioners, we concluded that the maximum level of investment in each stock was 0.1.

The fitness amounts for the objectives such as maximizing the portfolio return (n_1), minimizing the non-systematic risk (p_2), maximizing the stock return skewness (n_4), maximizing the level of portfolio liquidity (n_5) and maximizing the Sharp rate (n_6) in portfolio were measured as 10.371, 11.875, 0.875, 0.227 and 0.99, respectively through solving the single objective programming problems.

Since the normal beta in a market is 1 and the stocks whose beta is more than 1 are risky stocks and that the distribution of the return of such stocks is enormous; also, considering that those stocks whose beta is less than 1 are safe stocks and the distribution of their return is limited, accordingly we considered 1 as the fitness amount of systematic risk objective (p_3).

4.2. The Model's Response

All algorithms are coded in JBuilder and runs have been done on a Centrino 1.5GHz computer with 512MB memory. Also, the initial populations of all algorithms consist of random individuals. And each experiment (for each algorithm) was repeated 30 times with different random seeds. All algorithms run on equivalent conditions. Specific parameter settings of all algorithms are described in Table 2.

Table 2. Design parameters of GA and QPSO.

QPSO	Termination criterion(Max iteration)	150
	Size of the population	100
GA	Termination criterion(Max iteration)	150
	Size of the population	150
	Probability of crossover	0.8
	Probability of mutation	0.001
	Scale for mutations	0.1

The result of the application of QPSO is shown in Table 4. The presented results show the proportion of the stock which should be invested in the share of each company. For instance, 7.6 in the 10 the raw and the last column indicate that to minimize the fitness function The investor should invest 7.6% of his capital in the X_{49} variable (Behran oil company). In this section to measure the quality of the selected portfolio by QPSO, the return of the selected portfolio is compared to Markowitz Model and Genetic Algorithm. To compare the results we use the return of the portfolio within the 72 months time period ending in March 2008. Figure 3 shows the accumulated return of different portfolios within the time range in question. Figure 3 clarifies that the Markowitz portfolio gives a better return in comparison with multi-objectives model which has worked based on the genetic algorithm and QPSO.

Table 3. Optimal portfolio solutions.

Variable	Result	Variable	Result	Variable	Result
x_1	0.06	x_{18}	0	x_{35}	2.90
x_2	0.07	x_{19}	0.01	x_{36}	5.60
x_3	1.12	x_{20}	5.47	x_{37}	0.12
x_4	0.36	x_{21}	5.23	x_{38}	0
x_5	0.19	x_{22}	0.03	x_{39}	5.95
x_6	0.24	x_{23}	0.02	x_{40}	0.02
x_7	0.27	x_{24}	0.46	x_{41}	8.04
x_8	0.01	x_{25}	7.39	x_{42}	4.02
x_9	1.13	x_{26}	0.14	x_{43}	0.01
x_{10}	0.18	x_{27}	4	x_{44}	1.52
x_{11}	0.17	x_{28}	0.03	x_{45}	0.26
x_{12}	0.13	x_{29}	3.85	x_{46}	4.95
x_{13}	2.81	x_{30}	0.02	x_{47}	0.49
x_{14}	0.06	x_{31}	0.17	x_{48}	0.32
x_{15}	1.79	x_{32}	7.66	x_{49}	7.6
x_{16}	4.9	x_{33}	2.75	x_{50}	6.83
x_{17}	0.33	x_{34}	0.10		

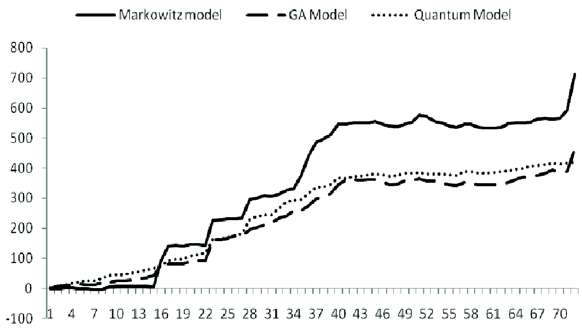

Figure 3. Ex-post performance of portfolios (return of portfolio).

Regarding the fact that investors counter with risk, mere reliance on return in the evaluation of performance is not a good criterion, although every investor prefer higher return, they also prefer not to risk. Therefore, to have a fair evaluation of portfolio performance we should determine whether the return is a function of risk? Hence, to relatively evaluate performance of the portfolios we should use methods of modified evaluation which take into account the amount of risk involved. Also, to check the variations of the portfolios we should use correlation coefficient and the number of the selected stocks. Table 3 illustrates the indicators for performance evaluation of the portfolios.

Table 4 shows that none of the models is superior to the other as far as all of the indicators are concerned. However, the multi-purpose model which has been solved by the use of QPSO has given highly Number of stocks in the portfolio (40) with less amount of non-systematic risk. Despite the fact that, the model solved by QPSO results in less return compared to Markowitz model, considering Trynor indicator it can be understood that the decrease in return has been coupled with less risk. Also, the modified indicators based on the risk confirm the superiority of the portfolios resulted from QPSO in comparison with two other models.

Table 4. Ex-Post performance of portfolios (competitions of models).

Model	Markowitz Model	GA Model	QPSO Model
Average Rate of Return	8.18	5.38	5.83
Standard Deviation of the Rate of Return (Nonsystematic Risk)	399.24	123.01	81.25
Jensen Coefficient (Free risk Return)	6.11	3.66	3.97
Treynor Ratio	3.6	3.81	3.26
Sharpe Ratio	0.34	0.37	0.5
Coefficient of Determination	0.18	0.19	0.47
Number of Stocks in the Portfolio	9	37	40
Number of Months with Negative Portfolio Return	26	20	12

Figure 4 shows the fitness value curve of the problem solved by the QPSO and GA after 400 iterations. The minimum fitness value of QPSO method is 3.47E-06 whereas the minimum fitness value of GA method is 3.67E-05. Figure 4 shows QPSO obtained better solution in shorter time.

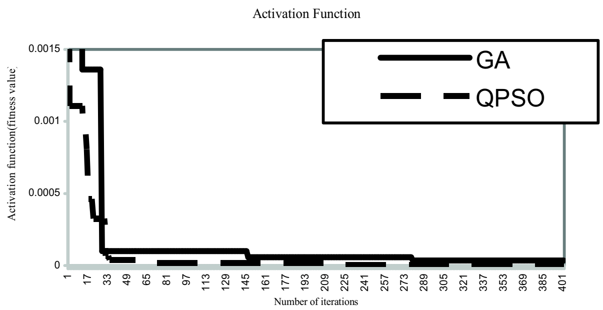

Figure 4. Activation function (fitness value) curve.

5. Conclusions and Discussions

Considering that the QPSO and GA models are non-linear and can be easily applied for numerous variables and can be upgraded effortlessly in case of adding a new variable (company) they can be claimed that these two models are more suitable model for selecting stock portfolio in comparison with Markowitz model. But finally in selecting between two artificial models regarding to indexes the fitness value, number of iterations, average rate of return, number of stocks in the portfolio and standard deviation of the rate of return, it could be concluded that the best model is QPSO in portfolio selection problem solving.

In this study, the objectives of the model were considered as explicit, nonetheless, in similar studies one can use fuzzy multi-purpose models in order to select a portfolio. Using this model one can take into account different dimensions of the reality of the issue, and in turn achieve more actual responses. Regarding the fact that the return of the stock is probable in nature, we suggest future researches apply probable multi-objectives in selecting stock portfolio.

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