Stability Coalition Formation with Cost Sharing in Multi-Agent Systems Based on Volume Discount

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Abstract: In Multi-Agent Systems (MAS), cooperation among agents to form coalitions based on volume discounts is a key topic. Such cooperation enables agents to achieve goals that they may not have been able to achieve independently at lower prices without ordering more than their real demand. In this paper, we propose a Stability Coalition Formation (SCF) and payoff distribution in terms of the core. Agents can enjoy a price discount for each of their requested action to achieve a goal through the concept of Social Agent Network (SAN), where different opportunities can be found. Each opportunity is associated with coalition value and search cost, given a search cost, the goal of the agent is to find the best set of opportunities which fulfills the coalition’s demands, along with a cost sharing rule satisfying certain stability properties. The experimental results illustrated that, the performance of proposed semi-optimal solution to SCF has proven its stability with average payoff 99.98% closest to the optimal payoff and higher than the average coalition value obtained by 9% when considered a search cost as a parameter affected on the search for optimal coalitions. Also, it has proven its efficiency in average processing time which saved and reduced by 15%~44% according to a different number of agents.

Keywords: MAS, CF, volume discount, search cost, stability.

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1. Introduction

In Multi-Agent Systems (MAS), where each agent has limited resources, Coalition Formation (CF) is a very powerful cooperation tool [20]. This cooperation is expressed with coalitions of agents that mostly desirable in environments, where a group of agents can perform a task more efficiently than any single agent can do alone.

In recent years, many CF models have been suggested for various domains such as E-Commerce [13, 21] where coalitions formed for the purpose of aggregating demands in order to obtain volume discount according to the size of the coalition (e.g., Tsvetovat et al. [19]), Cooperative Problem Solving (CPS) where group of agents are trying to form coalitions cooperatively to solve a problem (i.e., allocate tasks) and at the same time, strive to maximize its benefit while satisfying the goal and many other applications that have proven its importance [10]. However, the main problem can arise in these real world scenarios where, an agent may have limited abilities to obtain his goal individually. Thus, agents cooperatively make a coalition with partners extended from direct neighbors (agent’s neighbors) to indirect neighbors (neighboring agent’s neighbors) that give more opportunities for agents to obtain some or all of their goals at a discount price. In this paper, we consider a situation where agents are homogeneous in a sense that each agent has the same goal (to obtain an action at the lowest price) that follows this for CF and each agent benefits from joining a coalition because of the price discount incurred as a result, to achieve a stable payoff division. We call such CF as Stability Coalition Formation (SCF). Also, we assume that there are no available central mechanisms which can supply full immediate information of the entire system; thus the cooperation among agents is associated with a search cost. The search cost reflects the resources that need to be invested in search activities, such as locating other agents and determining the most valuable coalition. Thus, in the presence of the search cost, we propose a Coalition Value-based Strategy (CVS) to evaluate different opportunities during the search for SCF. Actually, this influences the tradeoff between continuing the search (possibly resulting in better opportunities by extending the coalition) or accepting the current CF (obtaining an immediate gain from the current coalition partners) according to the coalition value and payoff distribution. The main contribution of this paper is to present a semi-optimal solution algorithm for SCF problem that guarantees to reach a stable coalition (i.e., stable of payoff distribution within a coalition in terms of the core) through searching for feasible set of coalition partners in Social Agent Network (SAN) to obtain a volume discount.

The remainder of the paper is structured as follows. In the following section, we present a related work, and then we identify a formal description of SCF problem. We then develop a semi-optimal solution according to CVS. Building upon this, we present the experimental results and evaluation metrics of the proposed algorithm against other algorithms in the
related work and then we conclude the paper by discussing the most important features of our algorithm and proposing directions for future research.

2. Related Work

In many multi-agent environments, autonomous agents may find benefit in cooperation (i.e., forming coalitions) to achieve their goals and improve the overall performance [4, 7] when operating as a group. Aklouf and Drias [1] discussed the integration of MAS in electronic commerce architecture, where various agents interact together to resolve a problem.

Recently, many efforts have been done on the CF [12, 15] which involve self-interested agents and have achieved very great results [16]. Arib and Aknine [2] proposed a model based on the plans of the agents; they focus on self-interested agents operating in a system where the agents cannot reach their objectives individually. The majority of other research efforts focused on issues concerning the optimal allocation of agents into coalitions and the stability of such an allocation relies on the cost sharing rule. Yamamoto and Sycara [21] proposed a volume discount scheme for CF among buyer agents for electronic markets. In this scheme, they proposed a cost sharing rule in the core of optimal coalition by assuming incomplete information in the process. While, these algorithms relies on the availability of centralized agent that control the process and increase the value of total discounts but, none of them considered the cost or the network topology of agents in a decentralized manner. A Buyer Coalition Scheme with connection of a leader proposed in [3] which focused on some attributes from social networks. In this scheme, the coalition leader invited other members to join and form a new coalition by using his high level of centrality within the process. While, these algorithms relies on the availability of centralized agent that control the process and increase the value of total discounts but, none of them considered the cost or the network topology of agents in a decentralized manner. Therefore, we seek to form feasible coalitions and obtain volume discounts in a decentralized manner and at the same time, satisfying certain stability properties.

[5] Proposed a CF model within a fully decentralized MAS based on negotiation between agents and their direct neighbors only. Teistum [18] proposed an optimal coalition algorithm which induced by optimal subcoalitions and followed the main algorithm that discussed in [8]. These subcoalitions are formed of buyers requested the same item in a CF model. In that model, the first step is to sort all the buyers according to their reservation price for an item in descending order, generate all possible coalitions with regards to an item and then find the most valuable subcoalition among all possible ones to be considered as the optimal. While the completeness of these algorithms is an advantage in the sense that they guarantee to report the optimal solution within the payoff distribution in the core, there are some drawbacks since the solution space exponentially increased due to the increased number of agents in the search space for forming coalitions. The search activity is assumed to be costly in the real world environment where, agents do not know beforehand how many agents are willing to get the same coalition and determine the value of each potential coalition in order to choose the optimal one involves an excess of time. Therefore, it is necessary to present a solution which considered the search cost as a parameter affected on the formed of feasible and stable coalition.

The challenge is to find a semi-optimal solution algorithm for SCF that guarantees to obtain a stable solution when compared with optimal subcoalition algorithm [18] that is in stable and by considering the search cost as a parameter affected on the formation of optimal coalition (denoted as optimal-within-search cost).

3. SCF Problem

In this section, we provide a mathematical formulation of the SCF problem.

3.1. Formalization of the Problem

The general description of our proposed SCF problem assumes there a n number of agents \( A = \{a_1, a_2, \ldots, a_n\} \), set of task actions \( T_A = \{t_1, t_2, \ldots, t_m\} \) assigned to each agent, which represent his ability toward a given task. An agent is capable of performing only a subset of his task actions (i.e., sub-task) of a given task. We assume that, there is a Boolean function, \( \varphi \) from \( A \times T_A \) to \( \{true, false\} \), that associates with an agent \( a_i \) and action \( t_m \) the value true if the agent \( a_i \) is capable of performing an action \( t_m \) and the value false otherwise. Each agent specifies a cost function \( \beta: T_A \rightarrow \mathbb{R}^+ \), where \( \beta(t_m) > 0 \) indicates agent’s reservation price towards an action \( t_m \). For each action, there is a price function given by \( \delta(q): \mathbb{Z} \rightarrow \mathbb{R}^+ \); which \( \delta_i \) is the unit price of action \( t_m \) when \( q \) numbers of the requested action are provided together. The price function is a decreasing step function [14] which means it correlates the decrease of cost to the increase number of action requested, i.e., for \( q > q^* \), \( \delta(q) \geq \delta(q^*) \) and \( q < q^* \), \( \delta(q) > \delta(q^*) \). Since, an agent cannot perform all of the sub-tasks of a given task individually; it must cooperate with other agents in order to satisfy that task. Given this in mind, agents cooperate to form feasible coalitions and obtain discounts that maximize their payoffs. Stable Coalition \( C \geq t_m \) for an action \( t_m \) is a tuple \( \{C,\phi,\delta,\alpha\} \), \( C \) is a set of member agents whose \( \phi(\alpha, t_m) = false \); \( \alpha(\epsilon, C) \) is the coalition cost to obtain \( |C| \) numbers of \( t_m \); \( |C| \) is the cardinality of \( C \) (i.e., the term cardinality refers to the size of the coalition [17]); \( Z \) is the utility vector, \( U = \{u_1, \ldots, u_n\} \), where, \( u_i \) is the utility of agent \( a_i \) (i.e., the agent’s benefit from obtaining action \( t_m \) at cost \( c(t_m) \) by
joining a coalition) which takes the Quasi-linear Utility function as in Equation 1:

$$u_i = \beta(t_{m}) - c(t_{m})$$  \hspace{1cm} (1)$$

Where: $\forall a_i \in C_{t_m} : u_i \geq 0$

The gross value of a coalition $C$ is $V \in Z^+$; defined as surplus derived by serving the coalition as in Equation 2:

$$V(C) = \sum_{a_i \in C} (\beta(t_{m}) - cost_{t_{m}}(C))$$ \hspace{1cm} (2)$$

Therefore, the value of a coalition is equal to the sum of the utilities of the agents in the coalition; $V = \Sigma u_i$. The higher the value of $C$, the more efficient allocation of coalition $C$ to action $t_m$. In our problem, we consider that each requester agent asks for one unit of each requested action. In our solution, agents are trying to form coalitions with other agents that share the same task or sub-tasks to gain both the Individual Rationality (IR) and Budget Balance (BB) based on the volume discount scheme and cost sharing rule. Therefore, the goal is to find $C^*$ such that $V(C)$ is maximized as well as the resulting coalitions should be stable in the core as shown in Equation 3:

$$C^* = \arg \max_{C_{t_m} \subseteq \{A_i, i=1,..,m\}} V(C)$$ \hspace{1cm} (3)$$

3.2. Volume Discount Scheme

Assuming that agents have no capacity constraints on actions (each action is accompanied by a discount ratio) whose price ranged from minimum $DR_{min}$ to maximum $DR_{max}$ that the agent can provide based on the volume as shown in Figure 1. Table 1 describes a sample of discount rate based on volume.

![Volume discount scheme](image)

**Figure 1.** Volume discount scheme.

**Table 1.** Example of discount rate.

<table>
<thead>
<tr>
<th>Volume</th>
<th>Discount Rate</th>
<th>$\Delta d$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1-1</td>
<td>0.0</td>
<td></td>
</tr>
<tr>
<td>2-5</td>
<td>0.2</td>
<td></td>
</tr>
<tr>
<td>6-10</td>
<td>0.4</td>
<td></td>
</tr>
<tr>
<td>&gt;10</td>
<td>0.5</td>
<td></td>
</tr>
</tbody>
</table>

The volume column denotes the number of action requested to form a coalition and the discount rate column represents the ratio of the discount to the corresponding volume. According to the discount ratio, each agent within the coalition can obtain an action at a discount cost $D_{Cost_{t_m}}$ from its actual cost, where $D_{Cost_{t_m}} = \beta(t_{m}) \cdot \Delta d$ is the discount rate that defined based on the total number of action requested. Therefore, the coalition cost is $cost_{t_m}(C) = \beta(t_{m}) - D_{Cost_{t_m}}$

where, $\cos t_{m}(C)$ should satisfy that, $DR_{min} \leq cost_{t_{m}}(C) \leq DR_{max}$.

If agents cooperatively make a coalition, the price of obtaining an action is lower than that which is paid by an individual agent and based on the size of the final action’s coalition.

3.3. Agent’s Cost Sharing Rule

The stability of a coalition (i.e., Li et al. [9]) requires that the distribution of surplus among the coalition members is immune to groups of agents refusing to participate and form their own coalition, and the core is the most commonly used the concept to characterize this property. A general survey on the core of feasible allocations is provided in [6]. Therefore, it is important to determine the benefits that each agent should get in order to stay in a coalition such that the coalition may be considered to be stable (i.e., any deviation would never benefit these groups of agents, as their total payment if they remain in the coalition, is no greater than if they deviate). Stability relies on the cost sharing rule within each coalition. We thus adapt the cost sharing rule in [8] as well as a stability coalition obtained. A cost sharing vector of a feasible coalition, $C_{t_m}(x_{i,t} \leq C_i)$ specifies the cost $x_{i,t} \in Z^+$ to be paid by each agent $a_i$ in the coalition. A cost sharing vector is considered to be feasible if it satisfies the following constraints:

- **Constraint 1**: IR all individual payments are no greater than the corresponding reserve price (i.e., $\forall a_i \in C_{t_m} \cdot x_{i,t} \leq \beta(t_{m})$).

- **Constraint 2**: BB [11] the coalition is charged the cost incurred in the coalition (i.e., $\forall a_i \in C_{t_m} \cdot \sum_{a_i \in C_{t_m}} x_{i,t} \leq cost_{t_{m}}(C)$).

To ensure the stability of coalitions, we further impose the following condition on the cost sharing vector for each feasible coalition $C_{t_m}$ according to formula 4.

$$\forall \psi \in C_{t_m} : \sum_{a_i \in \psi} x_{i,t} \leq \delta_{C_{t_m}}(C)$$ \hspace{1cm} (4)$$

For any set of agent’s $\psi$ from the feasible coalition $C_{t_m}$, if they deviated from the coalition and formed a stand-alone coalition then, the unit price of the action $t_m$ will be at least $\delta_{C_{t_m}}(C)$. A cost sharing vector of the coalition that satisfied the condition 4 is said to be in the core of the coalition. Consider the cost sharing in which coalition members share cost as evenly as possible, is in the core with the following equation:

$$x_{i,t} = \begin{cases} \frac{hc_i}{c_i} & \text{if } (a_i \in \mathcal{C}_{i}) \\ \beta_i(t_{m}) & \text{if } (a_i \notin \mathcal{C}_{i}) \end{cases}$$ \hspace{1cm} (5)$$

Where, $hc_i = \left[ \sum_{a_i \in \mathcal{C}_{i}} z_{i,t} - cost_{t_{m}}(C) \right]$ and $c_i = c_i \cdot c_i - \beta_i(t_{m})$. Figure 2 shows the agents’ cost sharing rule including agent’s reservation price, its value share of cost and its actual price to pay. Agents that lie in $c_i \mathcal{C}_{i}$ pay $hc_i$, $hc_i \leq \beta_i(t_{m})$ and other agents that lie in $c_i \mathcal{C}_{i}$ pay just their reservation prices $\beta_i(t_{m})$. 
The agent’s cost sharing rule specified above is in the core of the coalition; which satisfied Equation 5.

Proof: According to the cost sharing rule, the agent's payment is the non-decreasing function of his reserve price. Therefore, we can prove that, the core of the coalition for any integer \( n \in \mathbb{N} \) we have

\[
\sum_{a \in a} x_{a} \leq \text{cost}_{a}(\psi_{a}) \quad \text{where}, \quad \psi(K) \text{ is the set of agents with } K \text{ highest reserve prices in} \ C_{a}, \text{with surplus derived by serving the coalition equals}
\]

\[
V(\psi_{a}) = V(\psi_{a}) - \sum_{a \in a} (\beta(\psi_{a}) \times \text{cost}_{a}(\psi_{a})).
\]

According to our proposed algorithm for SCF and CVS that described below, we have \( V(\psi_{a})) = V(\psi_{a}) \) for any \( K \). If \( \psi(K) \) deviated and form a stand-alone coalition for item \( t_{m} \) then, the value share of cost in the cost sharing rule is \( h_{c}(\psi_{a}(\kappa)) \) for \( \psi(K) \) and equivalent to prove that

\[
\forall a \in a \in (\psi(K)) \quad (h_{c}(\psi_{a}(\kappa)) \geq h_{c}).
\]

If \( \psi_{a} \subset c_{a} \) then, \( h_{c}(\psi_{a}(\kappa)) = h_{c}_{a} \), we can prove that by contradiction: Suppose \( h_{c}(\psi_{a}(\kappa)) < h_{c} \) then,

\[
h_{c}(\psi_{a}(\kappa)) < h_{c}. \quad \text{Hence, in this case,} \quad h_{c}(\psi_{a}(\kappa)) = h_{c}(\psi_{a}(\kappa)) \quad \text{and} \quad \sum_{a \in a} x_{a} \leq \text{cost}_{a}(\psi_{a}(\kappa)) \quad \text{Given,} \quad V(\psi_{a}(\kappa)) = \sum_{a \in a} \beta(\psi_{a}(\kappa)) - \text{cost}_{a}(\psi_{a}(\kappa)) \quad \text{it follows that the marginal value of} \quad c_{a} \quad \text{to} \quad c_{t}, \quad V(\psi_{a}(\kappa)) = V(\psi_{a}(\kappa)) = (\psi_{a}(\kappa)) - \text{cost}_{a}(\psi_{a}(\kappa)) \quad \text{which satisfies:} \quad V(\psi_{a}(\kappa)) = V(\psi_{a}(\kappa)) - \text{cost}_{a}(\psi_{a}(\kappa)) \quad \text{which is negative. This contradicts} \quad V(\psi_{a}(\kappa)) = V(\psi_{a}(\kappa)) - \text{cost}_{a}(\psi_{a}(\kappa)) \quad \text{which means that if the payoff division in the core of coalition, we say stable in the core (i.e., the state where the agents have no incentive to deviate from the coalitions to which they belong).}

4. Semi-Optimal Solution Algorithm

To allow agents form feasible stable coalitions given the formal description presented above, we have suggested that, agents are connected by SAN, \( \langle A, E \rangle \). It is an undirected graph, where vertices \( A \) are agents, and each edge \( e = (a_{i}, a_{j}) \in E \) indicates the existence of a social relation between agent \( a_{i} \) and agent \( a_{j} \). \( e = (a_{i}, a_{j}) \) means that, the agent \( a_{i} \) is a direct neighbor of agent \( a_{j} \) and in the \( a_{j} 's \) neighbor list \( Nlist(a_{j}) \). Let the set of neighbors (direct and indirect) for any agent denoted as \( \nu_{-} \subset V \). Each requested agent (i.e., \( a_{i} \in V \)) decides on actions it cannot perform alone and start searching for other requested agents (share the same requested action \( t_{m} \)) to form a group with them, \( T \subset V \) where, \( a_{i} \in T \), \( a_{j} \in a_{i} \) and \( \phi(a_{j}, t_{m}) = false \). Through searching process, the agent \( a_{i} \) tries to find also the most appropriate responder \( a_{j} \) that provides a discount price for the group of requested actions; \( \phi(a_{j}, t_{m}) = true \). Therefore, agent \( a_{i} \) can form feasible stable coalition \( C_{a} \) with other agents (not only with direct neighbors but, also with indirect neighbors) such that \( C_{a} \subseteq [a_{i}] \cup V \) for a specific action \( t_{m} \) with least cost (not only coalition’s cost \( cost_{a}(C) \) that defined in Equation 2 but also included a search cost and its influence over their search process as a part of the problem formulation). In this case, the proposed solution considered an additional cost \( \zeta \) which affected by some parameters as in Equation 6:

\[
\zeta = \tau_{res} \times \sum_{a \in a \in j = 1} \text{Nlist}(a_{j})
\]

\( \zeta_{res} \) is the size of coalition for a specific action; \( \tau_{res} \) is the responder’s level, which provide the action to the coalition’s members (this parameter reflects how the responder is far from the group of agents). In this case, the coalition value is the difference between the collective revenue (i.e., utilities of coalition’s members) and the search cost as seen in Equation 7:

\[
V(C_{a}) = \left( \sum_{a \in a \in} \beta(\psi_{a}(\kappa)) - \text{cost}_{a}(\psi_{a}(\kappa)) \right) - \zeta
\]

Thus, in the presence of the search cost, the agent uses CVS that guaranteed the feasibility of the solution for forming stability coalitions. According to CVS, the agent terminates the search when reaching a stable opportunity with current coalition value less than the previous one reached and at the same time the search cost at the previous search step is less than or equal to the search cost at the current search step. This strategy guarantees that, the agent can stop searching when at least found one responder agent and the utility of all members in the coalition is greater than or equal zero. At the same time, the stability condition should be satisfied (IR and BB). The procedure that describes the Semi-optimal solution algorithm illustrated below with a flowchart presented in Figure 3.
Input: The agent $a_i$’s goal $g_i$ (represent a task) and the set of requested actions $t_k$, $k=\{1, ..., m\}$ ($m$ represents the total number of sub-tasks for a given task). All actions should be satisfied to achieve a given goal.

Output: SCF (set of coalition partners) to achieve a specific goal with maximum coalition value and stable payoff distribution for coalition’s members.

The pseudo-code to get the solution for each action $t_k$ to satisfy a goal $g_i$, as follows:

1. Considering the requested agent $a_i$, as a singleton coalition for requested action $t_k$, $C_k = \{a_i\}$.
2. Searching in SAN for the semi-optimal solution that yields to a feasible stable coalition.
3. Finding the most valuable coalition that satisfied the constraint (the stable one with highest coalition value among reached ones) through the following steps:

   3.1. Get the least cost discount price $\pi = \pi(C)$ according to volume discount scheme for the current coalition whose coalition value $\nu(C_k)$ is maximized according to Equation 6, which accumulated with search cost $\zeta$ when agent $a_i$ asks another agent $a_j$ that located at one edge away from $a_i$ (i.e., direct neighbour $a_j \in Nlist(a_i)$) to perform an action $t_k$ in the goal $g_i$ and $\phi(a_i, t_k) =\text{true}$. In such a coalition, the coalition cost depends on the number of members within the coalition where: $\exists a_j \in A: \phi(a_j, t_k) =\text{false}$ and $c_{kj} = a_i \cup a_j$.

   3.2. According to the CVS, If $(\nu(C_k) < \nu^{\sim}(C_k)$ and $\zeta \geq \zeta^{\sim})$ then go to step 4, else $\forall Nlist(a_j)$ and $\phi(Nlist(a_j), t_k) =\text{true}$ (i.e., neighbouring agent’s neighbours) continue the process by going to step 3.1. Where: $\nu^{\sim}(C_{kj})$ is the current most valuable coalition value for that search stage associated with its search cost $\zeta$ and $\nu^{\sim}(C_{kj})$ is the most valuable coalition value from the previous search stage associated with its search cost $\zeta^{\sim}$.

4. Checking the payoff distribution in a stable manner according to the agent’s cost sharing rule where $\forall a_j \in C_k$ such that $u_{ij} \geq 0$ should be satisfied, go to step 5, instead go to step 3.

5. Stopping search and return the most valuable one as a feasible Stable coalition.

Searching for optimal coalition requires more extensive search especially, when the number of agents increase in the system. The number of possible coalitions potentially capable of carrying out a specific task, increase exponentially. Therefore, the proposed semi-optimal solution searches for the most valuable stable coalition (within one action) and stops when it is found according to the CVS (i.e., this is repeated for all the actions) by maximizing the value of feasible coalitions as well as should be stable in the core.

### 5. Experimental Results and Evaluation Metrics

To examine the dynamics of SCF and system analysis, a range of experiments were conducted with settings vary over the number of agents, the number of neighbors and the number of actions (sub-tasks) presented in Table 2. The experiments were implemented by Eclipse Java EE IDE V-1.2 running on Intel(R) core i3 CPU- 2.53 GHz with 8GB RAM (Windows 7) for achieving the set of feasible stable coalitions through random distribution of agents and associated neighbors in SAN that generated using uniformly distributed random numbers to follow the automated process for forming feasible coalitions. Each of these experiments repeated 100 times and the average of results are taken to ensure the accuracy and the efficiency of the proposed semi-optimal solution algorithm. The proposed semi-optimal solution algorithm compared with the optimal solution algorithm [18] and when that optimal associated with search cost (optimal-with search cost) to demonstrate the effectiveness of proposed semi-optimal solution in the presence of search cost during the formation of feasible discount coalition(s) in many real world systems.

<table>
<thead>
<tr>
<th>Exp. No.</th>
<th>No. of Agents</th>
<th>Max. no. of Neighbors</th>
<th>Total no. of Actions</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>20</td>
<td>4</td>
<td>2</td>
</tr>
<tr>
<td>2</td>
<td>40</td>
<td>8</td>
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</tr>
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</tr>
<tr>
<td>5</td>
<td>100</td>
<td>20</td>
<td>10</td>
</tr>
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</table>

To measure the efficiency and effectiveness of our proposed solution, we should consider the following metrics:
1. **Average Payoff Distribution**: It is the average payoff obtained from forming stable coalitions between agents.

2. **Average Processing Time**: This measure estimates the average time (in millisecond) taken to form coalitions. Reducing average processing time of CF is a very important performance measure.

3. **Average Coalition Value**: This measure estimates the global score of the system by computing the average of all coalition’s values.

In this paper, we use 95% confidence level to compute the confidence interval on the above three metrics that estimate the accuracy of the proposed solution. By considering that 95% confident that the population mean falls within the interval (sample statistic ± margin of error).

Figure 4 illustrated the confidence interval on average payoff for forming coalitions to obtain the goal. This figure reflects the stability of feasible coalitions that obtained from semi-optimal solution algorithm when compared with optimal stable one. It illustrated that, the average payoff that obtained from semi-optimal seems to be closest and better than the optimal-withsearchcost according to a different number of agents. Although, the optimal solution gives an optimal payoff distribution in a stable manner, the semi-optimal payoff results about 95%~99.98% closest to the optimal one. Therefore, the payoff distribution by using the cost-sharing rule resulted in stable feasible coalitions.

Due to the processing time for coalition formation, Figure 5 illustrated the efficiency of the proposed semi-optimal solution algorithm compared to optimal [18] and optimal-withsearchcost that considered the search cost for computing the optimality. Actually, searching as a group can reduce processing time for CF but in the same time, the average processing time (in millisecond) increased with respect to the increased number of agents in the system to be examined. In case of (optimal [18] and optimal-withsearchcost), the average processing time is very high that relies on searching the whole space for optimal coalitions.

According to different solutions presented in Figure 5, the processing time decreased by using the semi-optimal solution that find a solution in a reasonable time reduced by 15%~44% to allocate other requested agents in the coalition when compared with optimal-withsearch cost for achieving the goal. According to a different number of agents in Figure 5, the proposed solution has proven its efficiency for forming stable coalitions in time not less than 75% faster from the optimal. The confidence interval on average processing time computed and showed that semi-optimal solution algorithm is better than others in all experiments with respect to the margin of errors according to the CVS that control the search space especially when dealing with a large number of agent’s.

<table>
<thead>
<tr>
<th>No. of Agents</th>
<th>Semi-Optimal</th>
<th>Optimal</th>
<th></th>
<th></th>
<th>Semi-Optimal</th>
<th>Optimal</th>
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<td>3.7329</td>
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Figure 4. CI on average payoff.

The aim of CF is to maximize the agent’s payoff and coalition value and the aim of a volume discount is to get a price discount for obtaining the action with the lowest cost. Minimization of cost leads to maximization of total coalition value. By using volume discount scheme, agents can get a higher payoff by minimizing the cost for obtaining the action. Figure 6 shows the 95% confidence interval on average coalition value.

<table>
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Figure 5. CI on average processing time.

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Figure 6. CI on average coalition value.

Although, the optimal algorithm referred to highest coalition’s value according to Figure 6, the proposed semi-optimal solution algorithm has given average coalitions value about 93%~97% closest to the optimal.
one with different number of agents. The confidence interval on average coalition value proved that, forming coalition(s) with global score of the system is maximized and went up by 9% of the average coalition’s value obtained by optimal-with-searchcost solution.

6. Conclusions
This paper focused mainly on the opportunities for volume discount and in a stable manner by searching as groups through SAN. This leads to the concept of an agent having an incentive to form a coalition associated with the cost sharing rule to obtain a volume discount.

We adopt the weaker concept of the core of each coalition because the coalition structure core can only be defined for the optimal coalition configuration if the allocation is not optimal, agents can always improve their surplus by all deviating to the optimal one. According to CVS, the agents have no incentive to deviate from the coalitions to which they belong.

based on the experimental results, agents are able to form coalitions not only with direct neighbors but also with indirect neighbors (neighboring agent’s neighbors) to obtain their goals. Although, the optimal solution algorithm leads to optimal results in both the coalition value and payoff distribution, it takes high processing time especially when considered the search cost to reach the optimality. In contrast, the semi-optimal solution algorithm leads to payoff results that are very close to the optimal one and better than optimal-with-searchcost for a large number of agents in low average processing time. This makes the overall search cost is an important parameter affecting on the search for forming coalitions.

The SCF is suitable for real-world CF problems with a large number of agents toward specific goals to increase the coalition value and individual agent’s payoff especially when searching for stability that associated with search cost. There are many additional avenues of experimentation for dealing with other concepts of stability in the optimal solution; these will be left to future work.

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